Course Name: Functional Analysis **Course Code MAT602**

Credit: 4

Course Objective: The objectives of this course are as follows:

1: The main objective of this course is to introduce the concepts of Functional Analysis.

2: This course will provide the learners an opportunity to learn the basic Concepts and advanced concepts of Functional Analysis.

Normed and Banach spaces

Basics: Basic definition and result of Metric Space, Basic definition and result of Vector Space. Normed linear space, Further properties of Normed space, Extended Real Number System, Holder's Inequality for finite sequence, Minkowski's Inequality for finite sequences, Holder's Inequality for infinite sequence, Minkowski's Inequality for infinite sequences, Continuous at a point, Cauchy Sequence, Completeness, Banach Space Finite dimensional Normed Spaces, Equivalent norms, Compactness, F. Riesz's Lemma.

Linear Functional and Linear operator

Linear operator, bounded and continuous linear operator, linear functional, linear functional of finite dimensional spaces, Normed space of operators and dual space.

Inner product space and Hilbert space

Inner product spaces, Hilbert spaces and its example, Orthogonality, Orthonormal sets, Reisz Representation theorem, Legendre and Leguerre polynomial, Parsevals's theorem, the conjugate space of Hilbert space. Hilbert-Adjoint Operator, Self Adjoint, normal and unitary Operator, projection Operator.

Fundamental Theorems for Normed and Banach Spaces

Zorn's lemma, Hahn-Banach theorem and its applications, Adjoint operator, Reflexive spaces, Category Theorem: Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of sequence operators and functional, Open Mapping Theorem, Closed Linear Operator. Closed Graph Theorem, Banach Fixed Point Theorem.

Course Learner Outcomes

On successful completion of this course, learners will be able to:

1. Appreciate how functional analysis uses and unifies ideas from different and diverse area of mathematics.
2. Describe and apply fundamental theorems from the theory of normed and Banach spaces, including the Hahn-Banach theorem, parallelogram identity and Polarisation identity.
3. Recognize the role of Zorn's lemma.

.**UNIT SCHEDULE**

BLOCK I: NORMED, BANACH SPACES

1. Normed Space - I.
2. Normed Space - II.
3. Banach Space.
4. Finite dimensional Spaces.
5. Compactness and Finite Dimension.

BLOCKII: LINEAR FUNCTIONAL AND LINEAR OPERATOR

6. Linear operator.
7. Linear functional.

BLOCK III: INNER PRODUCT SPACE AND HILBERT SPACE

8. Inner Product space and Hilbert Space
9. Properties of Inner Product space and Hilbert Space
10. Hilbert-Adjoint Operator and other operator

BLOCKIV: FUNDAMENTAL THEOREMS FOR NORMED AND BANACH SPACES

11. Hahn-Banach theorem.
12. Category Theorem.
13. Open mapping theorem and Closed Graph Theorem
14. Banach Fixed Point Theorem

REFERENCES

1. E. Kreyszig, (1989), *Introductory Functional Analysis with applications*, John Wiley and Sons.
2. Walter Rudin, (1973), *Functional Analysis*, McGraw-Hill Publishing Co.
3. George F. Simmons, (1963), *Introduction to topology and modern analysis*, McGraw Hill Book Company Inc.
4. B. Chaudhary, S. Nanda, (1989), *Functional Analysis with applications*, Wiley Eastern Ltd.

SUGGESTED READINGS

1. H.L. Royden: *Real Analysis* (4^{th} Edition), (1993), Macmillan Publishing Co. Inc. New York.
2. J. B. Conway, (1990). *A Course in functional Analysis* (4^{th} Edition), Springer.
3. B. V. Limaye, (2014), *Functional Analysis*, New age International Private Limited.