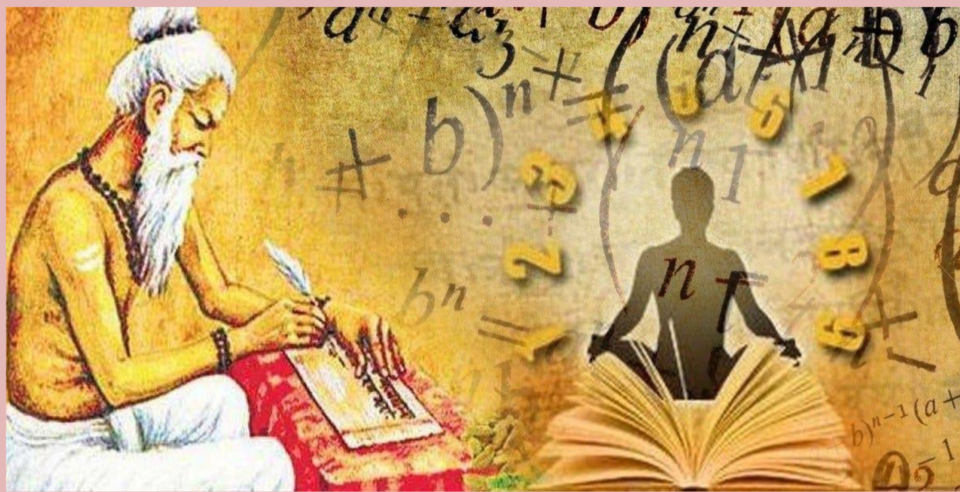
VEDIC MATHEMATICS VAC-12

FOR UNDER GRADUATE LEARNERS



DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCES
UTTARAKHANDOPENUNIVERSITY
HALDWANI, UTTARAKHAND
263139

COURSE NAME: VEDIC MATHEMATICS
COURSE CODE: VAC-12

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School of Science
Uttarakhand Open University
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CONTENTS

VEDIC MATHEMATICS

COURSE INFORMATION

The present self-learning material **"Vedic Mathematics"** has been designed for UG learners of Uttarakhand Open University, Haldwani. This course is divided into 10 units of study. This Self Learning Material is a mixture of Three Blocks.

The first block is **Introduction and History of Vedic Mathematics**. In the first unit of this block, we will study the great Indian mathematicians who contributed to the field of mathematics, such as Aryabhata, Brahmagupta, Bhaskaracharya, and others. Here, we explain how mathematics developed in ancient India and how the principles of mathematics spread from the Vedic period to the medieval period. In the second unit, we will study the great Indian mathematicians who contributed to the field of mathematics and how mathematics developed in ancient India. The third unit focuses on the life, work, and contributions of the great Indian mathematician **Srinivasa Ramanujan**. It explains his discoveries, his approach to numbers, and the similarities between his work and Vedic mathematics. Learners will understand how Ramanujan achieved worldwide fame in the field of mathematics despite having limited resources.

The second block is **Vedic Mathematics: Vedas, Śulva Sūtras, Baudhayana Theorem, and Indian Astronomy**. In the **fourth unit** of this block, we will study the mathematical concepts contained in the Vedas. The Śulva Sūtras provide concepts of geometric measurement, sacrificial altar construction, and proportion. Learners will understand how people in the Vedic era used mathematics for religious and scientific purposes. In the **fifth unit**, we introduce the theorems formulated by Baudhayana, including the Pythagorean Theorem. Learners will learn that Indian mathematicians explained theories related to triangles, area, and ratios long before Western mathematics. The **sixth unit** examines the development of Indian astronomy, highlighting the contributions of scientists such as Aryabhata, Varahamihira, and Bhaskaracharya, as well as the role of Vedic mathematics in planetary motion, time measurement, and calendar calculations.

Third block is Vedic Mathematics: Arithmetic Calculation**. The seventh unit** of this block is methods of quick calculation will be taught through Sutras. **Eight unit** is that how can we finding square roots and cube roots with the help of Vedic mathematics Learners will learn that Vedic methods are simpler and more time-saving than traditional methods. **Ninth unit** of this block will cover Vedic principles of algebra, such as linear equations, quadratic equations, and Vedic methods of factorization. Learners will understand how Vedic mathematics helps them solve algebraic problems mentally. The **tenth and last unit** will study the basic techniques of Vedic mathematics that are essential for higher mathematical studies, such as division of numbers, finding remainders (modular arithmetic), and the elementary principles of number theory.

The third block is **Vedic Mathematics: Arithmetic Calculation**. In the **seventh unit** of this block, methods of quick calculation will be taught through various Sūtras. The **eighth unit** explains how to find square roots and cube roots with the help of Vedic mathematics. Learners will understand that Vedic methods are simpler and more time-saving than traditional methods. The **ninth unit** covers the Vedic principles of algebra, including linear equations, quadratic equations, and Vedic methods of factorization. Learners will understand how Vedic mathematics enables them to solve algebraic problems mentally. The **tenth and final unit** focuses on the basic techniques of Vedic mathematics that are essential for higher mathematical studies, such as division of numbers, finding remainders (modular arithmetic), and the elementary principles of number theory.

Course Name: Vedic Mathematics **Credi**t-03
Course Code: VAC -12

SYLLABUS

1. **Introduction of Vedic Mathematics:** Definition of vedic mathematics, Historical background and origin of vedic mathematics, Principle of vedic mathematics, Benefit of vedic mathematics, Difference between conventional and Vedic mathematics, Development of concept of zero, concept of infinity, educational value of infinity in Vedic Maths, Contribution to modern mathematics
2. **History of Vedic mathematician:** Aryabhata's Life and work, Brahmagupta's life and Contributions. Bhaskara I Life and work, Madhava life and work, Mahaviracharya life and work, Bhaskara II life and work, Varahamihira life and work.
3. **Srinivasa Ramanujan:** Srinivasa Ramanujan (1887-1920). Brief outline of the life and mathematical career of Ramanujan. Hardy's assessment of Ramanujan and his Mathematics (1922, 1940).Some highlights of the published work of Ramanujan and its impact. Selberg's assessment of Ramanujan's work (1988). The saga of Ramanujan's Notebooks. Ongoing work on Ramanujan's Notebooks. The enigma of Ramanujan's Mathematics. Ramanujan not a Newton but a Mādhava.
4. **Mathematics knowledge in Vedas and Śulva Sūtras:** Mathematical references in Vedas. Comprehensive exploration of 16 Sutras and 13 upa(sub)-Sutras. The extant Śulbasūtra texts & their commentaries. The meaning of the word Śulba sūtra. Qualities of a Śulbakāra.
5. **Brief Introduction about Bodhāyana Theorem:** Finding the cardinal directions. Methods for obtaining perpendicular bisector. Bodhāyana's method of constructing a square. The Bodhāyana Theorem (so called Pythagoras Theorem) Applications of Bodhāyana Theorem. Constructing a square that is the difference of two squares.
6. **Indian Astronomy and Vedic Mathematics**: Continuing tradition of Indian Astronomy and Mathematics (1770-1870).
7. **Calculation with the help of Vedic mathematics:** Addition and Subtraction, Vinculum. Beejank (Reminder by Nine). Table. Mixed operations. Kaparekar constant. Multiplication (Nikhilam Sutra). Division of a number

8. **Techniques of root finding in Vedic mathematics:** Square. Cube. Division.Square root. Cube root. Divisibility. Ekadhikenpurven Method, Purven Method.
9. **Algebraic method in Vedic tradition:** Vedic methods to prime factorization. Divisibility rule. Modular arithmetic. Diophantine equation. Multiplication, (Urdhva-Tirayak Sutra). Mountain Expansion.
10. **Essential tool for higher arithmetic:** Power. Least common multiple. Recurring decimal. Solution of equations. Partial Fraction.

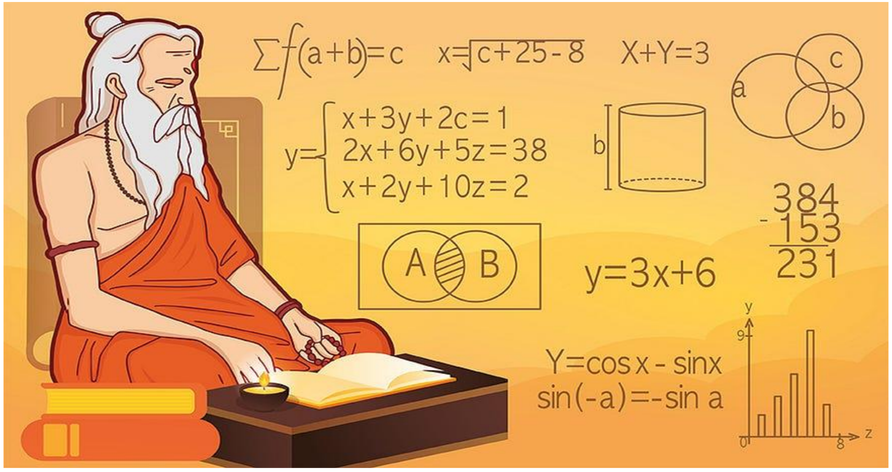
BLOCK I - INTRODUCTION OF VEDIC MATHEMATICS

UNIT I- WHAT IS VEDIC MATHEMATICS

CONTENTS:

1.1 INTRODUCTION

Vedic mathematics is an ancient and remarkable system of Indian mathematics based on the Vedas. It is a method of calculation that allows us to solve numerical problems in a simple, fast, and accurate manner. The principles of Vedic mathematics are mainly derived from the "Ganita Vedanga" of the Atharvaveda. The word 'Vedic' means "knowledge derived from the Vedas," and 'Ganit' means "the science of calculation or measurement." Thus, Vedic mathematics is the science that systematically presents the knowledge of calculation and numbers contained in the Vedas. Whereas normal mathematics often involves long and systematic procedures, Vedic mathematics provides a method for solving even complex calculations in a few easy mental steps. With the help of these formulas, problems of arithmetic, algebra, geometry, trigonometry, calculus and other complex branches of mathematics can be solved quickly and accurately. For example, operations like multiplication, division, square root and cube root, which take a lot of time to solve using the traditional method, are solved in just a few seconds through Vedic Mathematics.



Pic Credit: Federal news Fig1.1

Thus, Vedic mathematics is not just a calculation system but a complete system that combines ancient Indian knowledge with the needs of modern times. It serves as an effective tool for learners and researchers by making mathematics simpler, faster and more interesting. Vedic Math is a unique and powerful system of mathematics that offers many benefits, including improved calculation skills, increased self-confidence, and efficient problem-solving abilities. By understanding and applying Vedic math techniques, individuals can develop a deeper understanding of mathematics and improve their mathematical abilities.

In this unit we discuss about Historical background and origin of vedic mathematics, Principle of vedic mathematics, Benefit of vedic mathematics, Difference between conventional and vedic mathematics and Development of concept of Zero and Infinity.

1.2 OBJECTIVES

After studying this unit learner will be able

1. To understand the concept and foundations of Vedic Mathematics.
2. To explore the historical background and origin of Vedic Mathematics.
3. To identify the principles underlying Vedic Mathematics.
4. To analyze the benefits of learning Vedic Mathematics in modern education.
5. To distinguish between Conventional Mathematics and Vedic Mathematics.
6. To trace the development of the concept of Zero and Infinity in ancient civilizations and in Vedic thought.

1.3 DEFINITION OF VEDIC MATHEMATICS

Vedic Mathematics is an ancient calculation system of India, which is believed to have originated from the Vedas. It is based on 16 Sūtras and 13 Sub-Sūtras. With the help of these Sutras, even complex calculations can be solved quickly and easily. This method places special emphasis on mental calculation and logical thinking, thus making mathematics easy and interesting.Vedic Mathematics is characterized by its simplicity, quickness and mental calculation capability. It not only makes solving mathematical problems easier but also increases confidence in learners and reduces their fear of mathematics. Nowadays, this method is not only useful in the educational field but is also proving to be helpful in competitive examinations, computer science, cryptography and quick problem-solving techniques.

1.4 HISTORICAL BACKGROUND AND ORIGIN OF VEDIC MASTHEMATICS

Vedic mathematics originates from ancient Indian texts called the Vedas (c. 1500–500 BCE).The word Veda means "knowledge". Although there is no direct mention of mathematical formulas in the preserved Vedic texts, many mathematical ideas were contained in the Shūlbasūtras (a part of Vedic literature), which dealt with geometry, altar construction and measurement. In the early 20th century, Jagadguru Swami Bharati Krishna Tirthaji Maharaj (1884–1960) rediscovered and organised this knowledge. After deep study and contemplation of the Vedas, he compiled 16 Sūtras and 13 Upasūtras. These gave a simple but powerful system of calculation. Thus, while Vedic mathematics has its roots in the Vedic tradition, its modern structured form was presented by Swami Bharati Krishna Tirthaji in his famous book “Vedic Mathematics” (published posthumously in 1965).

Roots in Vedas: The basis of Vedic mathematics is derived from the ancient Vedas, a collection of Indian scriptures that are thousands of years old. The Vedas, which means "knowledge", cover a variety of subjects, including mathematics, philosophy, and astronomy. The Shūlba Sūtras, a part of Vedic literature, describes mathematical concepts and methods for geometrical calculations, altar construction and measurement. These ancient texts reveal a deep understanding of mathematical principles, developed as a practical and spiritual pursuit.The connection to the Vedas provides Vedic mathematics with a rich historical and cultural context, reflecting the importance of mathematical knowledge in ancient Indian traditions.

Atharva Veda: Of the four Vedas, the Atharva Veda is considered to be the oldest source of Vedic mathematical principles. In this, many Sūtras and Mantras are found which contain indications related to numbers, calculation methods, and measurement. Although these formulas are not in modern mathematical language, they reflect the basic concepts of arithmetic and geometry. For example, the methods of measurement, angles and proportions used in altar construction and sacrificial ceremonies are extremely important from a mathematical point of view. Thus, the Atharva Veda is considered a key basis for understanding the historical foundations of Vedic mathematics. The main medium of education in the Vedic period was Shruti and Smriti, that is, knowledge was transmitted from the Guru to the disciple through oral tradition. The methods of Vedic mathematics were also transmitted from generation to generation through this oral tradition. These were considered part of the Vedangas (six auxiliary disciplines related to the study of Vedas — Shiksha, Kalpa, Vyakarana, Nirukta, Chhanda and Jyotish). Jyotish Vedanga, in particular, contained mathematical principles related to calculation, astronomy and time measurement.

1.5 PRINCIPLE OF VEDIC MATHEMATICS

Vedic mathematics is an ancient mathematical system, the basis of which are 16 main Sūtras and 13 Sub-sūtras. All these Sūtras are short sentences in Sanskrit, in which deep mathematical knowledge is presented in a concise and simple form. The principles of Vedic mathematics are based on the following features:

(i) **Speed and accuracy:** Vedic methods make calculations extremely fast and give accurate results. Especially complex calculations like multiplication, division, square root and cube root can be solved in seconds.

(ii) **Versatility:** The same formula can be used for different types of problems. For example, the "vertical-oblique" formula can be used in multiplication, algebra, and geometry.

(iii) **Simplicity and Briefness:** All the formulas of Vedic mathematics are short and easy to remember. Using them, even difficult and long calculations can be solved quickly mentally.

(iv) **Mental calculation:** The biggest principle of Vedic Mathematics is that calculations can be done mentally without using pen and paper. This develops confidence and the ability to take quick decisions in learners.

(v) **Creativity and reasonableness:** Vedic Mathematics is not just a calculation technique, but it also encourages logical thinking and creative approach. This allows learners to learn Mathematics joyfully rather than considering it a difficult subject.

1.6 BENEFIT OF VEDIC MATHEMATICS

Vedic Mathematics is not just a method to simplify calculations, but it also provides many benefits to learners and math learners. Its main benefits are as follows:

(i) **Fast and Easy Calculations:** Vedic math's makes calculations very fast. It becomes possible to solve long calculations mentally in just a few seconds.

(ii) **Development of Mental Ability:** This system is based on mental calculation, which develops memory, concentration and reasoning power.

(iii) **Increase in confidence:** The ability to solve difficult problems quickly and correctly builds confidence in learners.

(iv) **Reduces Chances of Errors:** Formula based calculation method reduces mathematical errors and gives more accurate results.

(v) **Useful in All Branches of Mathematics: Vedic** mathematics can be used in subjects like arithmetic, algebra, geometry, trigonometry and calculus.

(vi) **Helpful in Competitive Exams:** The ability to solve mathematical problems quickly is very helpful in competitive exams like Banking, SSC, UPSC, CAT etc.

(vii) **Encourages Creative and Logical Thinking:** Vedic Mathematics encourages learners to solve problems in different ways, thereby enhancing their creativity and logical approach.

(viii) **Removes Fear of Mathematics:** It makes Mathematics, which is considered a difficult subject, simple and interesting, thereby eliminating the fear of Mathematics from the learners.

Pics credit :Winaum learning, Fig.1.2

1.7 DIFFERENCE BETWEEN CONVENTIONAL AND VEDIC MATHEMATICS

Aspect	**Conventional mathematics**	**Vedic mathematics**
Origin	Developed over centuries through contributions from Greek, Indian, Arabic, and modern mathematicians.	Based on **16 sutras (formulas)**and **13 sub-sutras** rediscovered by Swami Bharati Krishna Tirthaji from ancient Vedic texts.
Approach	Step-by-step, lengthy, rule-based methods for solving problems.	Uses **shortcuts, mental methods, and pattern recognition** for quick solutions.
Calculation Speed	Slower, especially for large or complex numbers.	Much faster due to simple and direct technique

Complexity	Can become complicated with lengthy formulas and multiple steps.	Simplifies problems into easy, manageable steps.
Mental Involvement	Relies more on written work and calculators.	Encourages **mental calculations,** enhancing concentration and memory.
Application	Widely used in education, engineering, science, finance, and technology.	Useful for **fast arithmetic, competitive exams, algorithm design, cryptography, and computer science.**
Global Recognition	Universally accepted as the standard system of mathematics.	Gaining recognition worldwide as an **alternative and complementary system**.
Learning Experience	Sometimes creates "math fear" among learners due to difficulty.	Makes mathematics more **engaging, fun, and less intimidating.**

1.8 DEVELOPMENT OF CONCEPT OF ZERO

The **mathematical zero ("0")** — used as a number with its own identity — was first clearly defined in India around the 5th to 7th century CE. The Indian mathematician **Brahmagupta** (628 CE) is credited with:Defining zero as a number, developing rules for arithmetic involving zero (e.g., addition, subtraction. Mathematician **Brahmagupta** (598 – 668 CE) outlined rules for arithmetic involving zero in his work *Brahmasphuṭasiddhānta*, marking a pivotal development in the history of mathematics.

In Vedic Mathematics Zero's concept is implicit for calculations. Although the modern symbol for zero wasn't used, the idea of "nothing" or "shunya" (Sanskrit for zero) was acknowledged. Vedic Mathematics uses a place value system, which relies on the concept of zero as a placeholder. Zero is used in calculations, such as multiplication and division, to represent the absence of quantity.

Some Vedic Mathematics sutras, like the **"Ekadhikena"** sutra, involve calculations that implicitly use the concept of zero. Zero is also associated with spiritual concepts, such as the infinite and the universe.

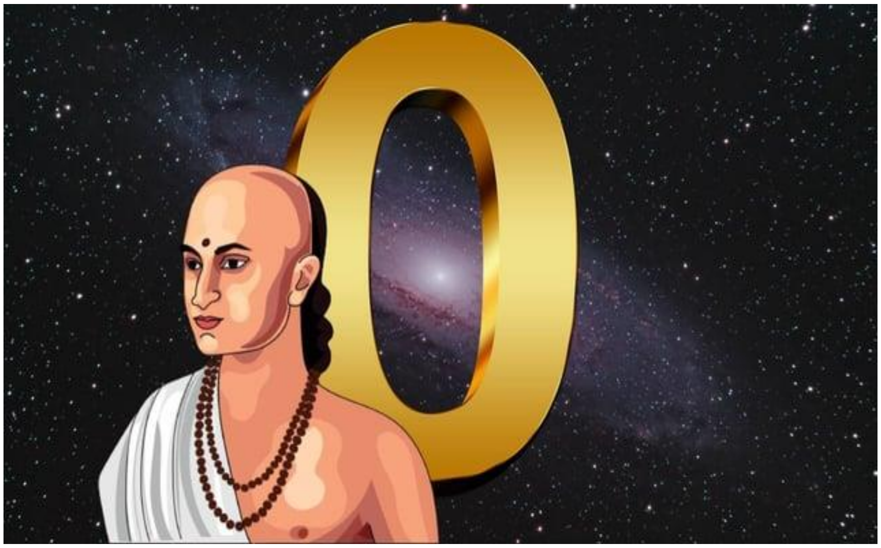


Image Credit: Moralinefo.Com Fig.1.3.

The concept of zero in Vedic Mathematics is rooted in ancient Indian understanding of mathematics and philosophy. While the modern symbol wasn't used, the idea of zero was integral to calculations and philosophical discussions.

Mention of Zero in Vedic Literature: The roots of Vedic mathematics are found in the Rig Veda and Atharva Veda, where the philosophical concept of "Zero" or "Void" is present. In the Upanishads, zero is described as a symbol of both "complete" and "incomplete".

Pingalacharya (2nd century BC): Pingalacharya used "zero" in prosody. He used place-value system in calculations, which made the importance of zero clear.

Aryabhata (5th century): Aryabhata systematized the place-value system, where zero served as a place holder for digits. Although he did not create the symbol "0", he applied the concept of it in calculations.

Brahmagupta (7th century): Brahmagupta defined zero as an independent number in his work "Brahmasphutasiddhanta". He gave mathematical rules related to zero:

(i) If you add zero to a number, the number remains the same.
(ii) If you subtract zero from a number, the number remains the same.
(iii) Multiplying a number by zero gives the result zero.
(iv) However, the solution to the problem of division by zero was developed later.

Bhaskaracharya (12th century) : Bhaskaracharya explained the use of zero in more depth. He considered the problem of division by zero and linked it to infinity.

Importance in Vedic Mathematics:

(i) Zero completed the decimal system.
(ii) Many methods of Vedic mathematics are based on this concept of zero.
(iii) Today's modern mathematics, computer science and technology are not possible without zero.

The concept of zero is India's gift, which developed from Vedic literature to Aryabhata, Brahmagupta and Bhaskaracharya. In Vedic mathematics, zero is not just an empty space but the soul of mathematics, which gave a new direction to the numerical system and modern mathematics.

1.8.1 ORIGIN IN ANCIENT CIVILIZATION

Mesopotamia (Sumerians): Around 5,000 years ago, the Sumerians used a placeholder symbol—a slanted double wedge — in their cuneiform numeral system to denote the absence of a value in a position. This was an early step toward the concept of zero as a placeholder

Babylonians: By approximately 400 BC, Babylonians employed two wedge symbols to indicate the absence of a digit in a place value, distinguishing numbers like 216 from 21 6.

Babylonians: By around 400 BC, Babylonian scribes used a placeholder symbol made of two small wedge marks to indicate the absence of a digit in a sexagesimal (base-60) place-value system. This helped distinguish numbers such as 216 from $21 \times 60 + 6$**,** though the symbol was not a true zero and was not used at the end of numbers.

1.8.2 DEVELOPMENT IN ANCIENT INDIA

India Brahmagupta (7th Century CE): The Indian mathematician Brahmagupta was the first to formalize the use of zero as a number. He introduced rules for arithmetic operations involving zero, treating it as a number rather than merely a placeholder. He referred to zero as "shunya," meaning void or empty.

1.8.3 TRANMISSION TO THE ISLAMIC WORLD AND EUROPE

Islamic Mathematicians: The concept of zero spread to the Islamic world, where scholars translated and expanded upon Indian mathematical texts, further developing the concept.

Fibonacci (13th Century CE): The Italian mathematician Fibonacci introduced the Hindu-Arabic numeral system, including zero, to Europe through his work "Liber Abaci," facilitating its adoption in Western mathematics.

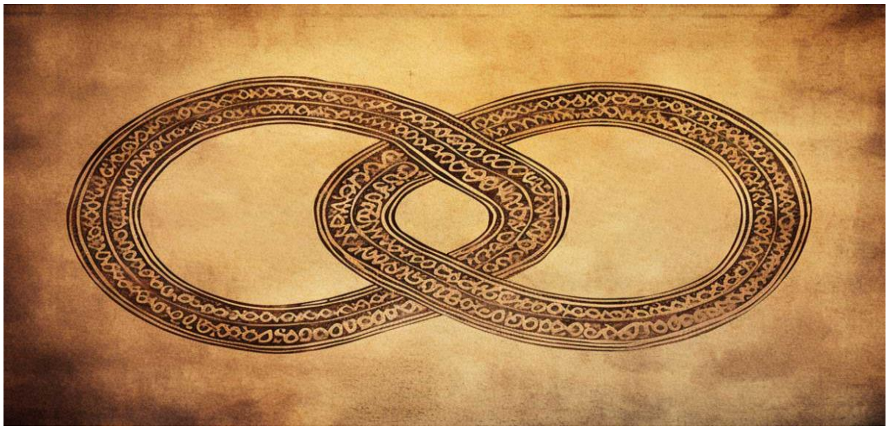
1.8.4 SIGNIFICANCE IN MATHEMATICS AND BEYOND

Placeholder and Number: Zero serves both as a placeholder in positional numeral systems and as an independent number, enabling the representation of large numbers and the development of advanced mathematics.

Foundation for Modern Mathematics: The acceptance of zero was crucial for the development of algebra, calculus, and the binary system, which underpins modern computing.

1.9 CONCEPT OF INFINITY

Infinity (∞) is a very profound and philosophical concept in mathematics, which means - "that which has no end" or "boundless". In another words it is often associated with the Sanskrit term “Ananta” means infinite or endless. It is discussed in the context of the universe’s vastness and cyclical nature. The idea of infinity is very ancient in the Indian tradition and is described in the Vedas, Upanishads and Vedic mathematics. Infinity is not merely a very large number—it represents total completeness beyond all limitations, a fundamental idea in both spiritual philosophy and mathematical understanding.



Concept of infinity in Hindu framework (prachyam.org) Fig 1.4.

Ananta in Vedas and Upanishads: In Ishavasya Upanishad, Ananta is described as follows:

"पूर्णमदः पूर्णमिदं पूर्णात् पूर्णमुदच्यते।
पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते॥"

Which Means: Even if you subtract infinity from infinity, infinity still remains. Here infinity is considered the symbol of Brahma (the ultimate element).

Early mathematical usage: In Vedic mathematics, "infinity" was used in situations where numbers became extremely large. Pingalacharya (2nd century BCE) mentioned infinite possibilities in Chhandashastra.

Bhaskaracharya (12th century): In his works "Lilavati" and "Bijganita", Bhaskaracharya gave a mathematical definition of infinity. He said, dividing any number by zero gives infinity. He explained infinity not as a "special number" but as a "limitless quantity".

Kerala School and Madhvacharya (14th century): Maths of Madhvacharya and the Kerala School worked on infinite series. They developed trigonometric series such as π (pi), sine and cosine, which later became the basis of calculus.

Importance in Vedic Mathematics: Ananta showed that computation is not limited to a finite number of numbers, but extends to infinite possibilities. This concept was fundamental in the development of astronomy, geometry, algebra and modern mathematics.

1.9.1 PHILOSOPFICAL FOUNDATION (VEDIC LITERATURE*)*

The **Rigveda** and **Upanishads** describe the universe as *Ananta* (अनन्त), meaning **endless or infinite**. The **Isha Upanishad** beautifully expresses this idea:

पूर्णमदः पूर्णमिदं **पूर्णात् पूर्णमुदच्यते ।** **पूर्णस्य पूर्णमादाय** **पूर्णमेवावशिष्यते ॥**	*वह पूर्ण है, यह भी पूर्ण है।* *पूर्ण से ही पूर्ण उत्पन्न होता है।* *पूर्ण में से पूर्ण को लेने पर भी* *पूर्ण ही शेष रहता है।*	pūrṇam adaḥ pūrṇam idaṁ pūrṇāt pūrṇam udacyate | pūrṇasya pūrṇam ādāya pūrṇam evāvaśiṣyate ||

"That is boundless, this is boundless; from the boundless, the boundless arises..."
Infinity is not merely a very large number—it represents total completeness beyond all limitations, A fundamental idea in both spiritual philosophy and mathematical understanding.

CHECK YOUR PROGRESS

True and False

1. Zero was first used as a placeholder in ancient Babylonian numerals. – **True**
2. The symbol "0" was first fully developed in India. – **True**
3. The concept of zero was known in Europe before it appeared in India. – **False**
4. Aryabhata used zero as a number with mathematical operations. – **False** (He used place-value system but not the symbol 0)
5. Brahmagupta defined rules for operations involving zero. – **True**

MULTIPLE CHOICE QUESTIONS

1.Vedic Mathematics is mainly derived from which Veda?

a) Rigveda
b) Samaveda
c) Yajurveda
d) Atharvaveda

2. **Who rediscovered Vedic Mathematics in modern times?**

a) Aryabhata
b) Bhaskara
c) Swami Bharati Krishna Tirthaji Maharaj
d) Brahmagupta

3. T**he concept of Zero originated in which civilization?**
a) Greek
b) Indian
c) Egyptian
d) Chinese

4. **Who first defined Zero as a number?**
a) Aryabhata
b) Bhaskaracharya
c) Brahmagupta
d) Pingala

5. **How did the Indian numeral system reach Europe?**
a) Through the Silk Route
b) Through Arab scholars
c) Through Greek translations
d) Through the Roman Empire

6. **What is the importance of Zero in Mathematics?**
a) Just a symbol
b) Makes calculations difficult
c) Foundation of the place value system
d) No importance

7. **The verse 'Pūrṇam adaḥ pūrṇam idam' represents the philosophical basis of which concept?**
a) Zero
b) Infinity
c) Decimal system
d) Multiplication method

8. **The concept of Infinity first appeared in which tradition?**
a) Greek
b) Vedic
c) Roman
d) Egyptian

9. **How was the concept of Infinity studied in ancient Indian mathematics?**
a) Through physical experiments
b) Through infinite series and limits
c) Only as a religious idea
d) It was never studied

10.**What is the educational value of Infinity in Vedic Mathematics?**
a) Only used in calculations
b) Develops creative and abstract thinking among learners
c) Makes learning difficult
d) Used only for religious teaching

Fill in the Blanks:

1. The earliest use of zero as a placeholder is found in ________.
   Answer: Babylonian civilization

2. The symbol "0" was first written in ancient ________.
   Answer: India
3. The mathematician who gave rules for arithmetic operations on zero was ________.
   Answer: Brahmagupta
4. The concept of zero spread to Europe through ________ mathematicians.
   Answer: Islamic / Arab
5. Zero plays a central role in the ________ numeral system.
   Answer: Place-value

1.10 SUMMARY

Vedic mathematics is a unique mathematical system from ancient India, whose origins are traced back to the Vedas, particularly the Atharvaveda. It was rediscovered in the 20th century by Jagadguru Swami Bharati Krishna Tirthaji Maharaj. Vedic Mathematics has 16 Sutras and 13 Sub-Sutras, with the help of which complex mathematical problems can be solved simply, quickly and mentally. Vedic Mathematics is a system of mathematical techniques and principles rooted in ancient Indian texts, known for its simplicity, speed, and efficiency. It provides shortcut methods for calculations in arithmetic, algebra, geometry, and more, making it particularly useful for competitive exams and mental math.

Zero and infinity: Zero and infinity were not just mathematical innovations in India—they were deeply philosophical concepts that evolved into practical tools over centuries.

Place-value system: It is a method of representing numbers in which the position of each digit determines its value. This system is fundamental to modern arithmetic and numeration, and its origin can be traced back to ancient India.

1.11 GLOSSARY

- **i.** **Vedas** – Ancient sacred texts of India containing spiritual, philosophical, and scientific knowledge.
- **ii.** **Vedic Mathematics** – A system of mathematics derived from the Vedas, based on simple mental calculation techniques using Sutras.
- **iii.** **Sutra** – A short Sanskrit formula or aphorism that provides a quick mathematical method.
- **iv.** **Upa-sutra** – A sub-formula used to support or extend the main Sutra in Vedic Mathematics.
- **v.** **Atharvaveda** – One of the four Vedas; the main source from which Vedic Mathematics is believed to have originated.
- **vi.** **Jagadguru Swami Bharati Krishna Tirthaji Maharaj** – The 20th-century scholar who rediscovered and compiled Vedic Mathematics.
- **vii.** **Conventional Mathematics** – The modern system of mathematics taught through step-by-step written methods.

viii. **Zero (0)** – A number representing 'nothing'; a symbol that plays a key role in the place value system and modern arithmetic.

ix. **Infinity (∞)** – A concept representing something that has no limit or end; used in mathematics and philosophy.

x. **Ananta** – Sanskrit term meaning "endless" or "infinite"; used in Vedic literature to describe infinity.

xi. **Pūrṇam adaḥ pūrṇam idam** – A verse from the *Isha Upanishad* that expresses the concept of infinity and completeness.

1.12 REFERENCES

1. Baladev Upadhyaya, *Samskrta-Śāstromka-Itihās*, Chowkhambha,Varanasi,2010.
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3. D.M.Bose, S.N.Sen and B.V. Subbarayappa, Eds., *A Concise History of Science in India*, 2nd Ed., Universities Press, Hyderabad, 2010.
4. *Astāngahrdaya*,Vol.I, *Sūtrasthāna* and *Śarīrasthāna*, Translated by K. R. Srikantha Murthy, Vol. I, Krishnadas Academy, Varanasi, 1991.

1.13SUGGESTED READING

1. "Vedic Mathematics" by Bharati Krishna Tirthaji, The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras. Covers mental math techniques for arithmetic, algebra, and geometry
2. "Vedic Mathematics Made Easy" by Dhaval Bathia, A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.
3. "The Power of Vedic Maths" by Atul Gupta, Focuses on speed and accuracy. Ideal for practical application in exams like CAT, GRE, and banking tests.
4. "Mathematics in India" by Kim Plofker, A scholarly account of the historical development of mathematics in India.Provides context on how Vedic methods evolved over time.
5. "Ancient Indian Leaps into Mathematics", B. S. Yadav and Manju Bhargava Covers Vedic and post-Vedic developments with examples and explanations.

1.14 TERMINAL QUESTIONS

1. Define **Vedic Mathematics** in your own words.

   ………………………………………………………………………..

2. Explain the **historical background and origin** of Vedic Mathematics.Mention any **two main principles** of Vedic Mathematics.

   ………………………………………………………………………………………

3. Differentiate between **Conventional Mathematics** and **Vedic Mathematics**.

   ………………………………………………………………………………………….

4. Who first defined **Zero as a number** and what was its significance?

   ……………………………………………………………………………………………

5. Trace the **development of the concept of Zero** from ancient civilizations to modern times.

   ………………………………………………………………………………………………….

6. How do the **concepts of Zero and Infinity** reflect India's contribution to world mathematics?

   ……………………………………………………………………………………………..

1.15 ANSWERS

CYQ 1. True **CYQ 2.** True **CYQ3**. False **CYQ4.** False **CYQ5.** True

Fill In the blanks
1.Babylonian civilization 2. India 3. Brahmagupta 4. Islamic / Arab 5. Place-value

MCQ 1. D **MCQ 2**. C**MCQ 3**. B**MCQ 4.** C **MCQ 5**. b
MCQ 6. C**MCQ 7.** B **MCQ 8**. B**MCQ 9.** B**MCQ 10. b**

UNIT 2 –HISTORY OF VEDIC MATHEMATICS

CONTENTS:

2.1 INTRODUCTION

Mathematics in India has a very ancient and rich history. During the Vedic period (1500 BCE to 500 BCE), mathematics was involved in every aspect of life—the construction of sacrificial altars, agriculture, astrology, time measurement, and architecture all involved mathematics. Ancient texts such as the Shūlba Sūras, Baudhayana, Apastamba, Katyayana, and the Maunaka Sutras mention mathematical principles. For example: The Baudh**ā**yana Sutras contain the description of the Pythagorean Theorem, which was discovered much later in the Western world. Pingala's Prosody uses binary numbers, which are the basis of modern computer science. India has a long tradition of mathematical innovation. After the classical period of mathematicians like Aryabhata and Brahmagupta, the medieval era (6th–15th centuries CE) produced other brilliant minds who made ground breaking contributions in arithmetic algebra, trigonometry, and astronomy. India has a long tradition of mathematical innovation. After the classical period of mathematicians like Aryabhata and Brahmagupta, the medieval era (6th–15th centuries CE) produced other brilliant minds who made ground breaking contributions in arithmetic, algebra, trigonometry, and astronomy.

Dear learner previous unit we have studied the topics: Historical background and origin of vedic mathematics, Principle of vedic mathematics, Benefit of vedic mathematics, Difference between conventional and vedic mathematics and Development of concept of Zero and Infinity. This unit explores the lives and works of four such luminaries: Bhāskara I Bhāskara II, Varāhamihira, and Madhava of Sangamagrama.

2.2 OBJECTIVES

After studying this unit learner will be able

1. To learn about the life of great mathematicians of ancient India, Aryabhata and Brahmagupta.
2. To understand the important contribution of Aryabhata and Brahmagupta.
3. To Explain the contributions of Bhāskara in fields such as arithmetic, algebra, geometry, and astronomy.
4. Understand the lives and achievements of Bhāskara II, Varāhamihira, and Madhava.
5. Appreciate the historical context of medieval Indian mathematics.
6. Recognize India's contributions to trigonometry, algebra, and early calculus concepts.

2.3 ARYABHATA'S LIFE AND WORK

Aryabhata was one of the greatest mathematicians and astronomers of ancient India. Aryabhata was born in Patna, Bihar, India, during the Gupta dynasty. While the exact birthplace is unknown, it is widely believed that he was spent his life in Bihar, specifically in the region around Kusuma Pura. Little is known about his personal life, but his works suggest he was well-versed in mathematics and astronomy. Aryabhata is believed to have studied at the ancient university of Nalanda in Bihar. He later became the head of an astronomical observatory or research Centre in Kusuma Pura (Modern day Patna). Not much is known about his personal life, and historical records about the personal life of Aryabhata are sparse and limited. While the absence of personal details leaves much unknown about Aryabhata's daily life, his enduring legacy lies in the impact of his groundbreaking ideas and theories in the fields of mathematics and astronomy. Aryabhata is revered as a pioneering scholar, and his contributions have earned him a lasting legacy. There are various educational institutions and awards named in his honor in recognition of his profound impact on mathematics and astronomy. While Aryabhata's exact biography remains somewhat elusive, his mathematical and astronomical treatises have left an indelible mark on the history of science and continue to be studied and celebrated for their intellectual rigor and groundbreaking ideas.

The exact details of **Aryabhata's death** are not clearly known, as historical records from that time are limited. However, it is generally believed that he died around **550 CE**, possibly in **Kusuma Pura** (modern-day Patna, Bihar), where he is thought to have spent much of his life.Since there is no direct historical evidence describing the circumstances or exact date of his death, most of what we know about Aryabhata comes from his writings and the references made by later scholars. Despite the lack of detailed records about his death, Aryabhata's contributions to mathematics and astronomy have ensured that his name lives on as a monumental figure in Indian scientific history.

Aryabhata was a pioneering Indian mathematician and astronomer whose most famous work is the **Āryabhaṭīya**, written in **499 CE** when he was just 23 years old. This Sanskrit text is a compact but highly advanced treatise that laid the foundation for many areas of mathematics and astronomy in India and beyond.

ARYABHATA'S WORK

Aryabhata's most renowned work is the "**Ā**ryabhat**ī**ya," a comprehensive text that covers various aspects of mathematics and astronomy. The **Ā**ryabhat**ī**ya is composed of 118 poetic verses that delve into a wide range of mathematical disciplines such as arithmetic, algebra, trigonometry, and astronomy.In this seminal work, Aryabhata showcased his advanced understanding of time measurement. Additionally, he defined a day as comprising 24 hours and provided observations on how the lengths of day and night change depending on geographic latitude. Aryabhata played a pivotal role in advancing the understanding of numbers and their characteristics. He was an early pioneer of the decimal system and recognized the importance of zero long before it gained prominence elsewhere. In algebra, he tackled quadratic equations and devised effective techniques for solving them. His work in trigonometry was equally influential—he introduced the sine function and compiled precise trigonometric tables that would later serve as a foundation for astronomical studies. Aryabhata also made ground-breaking strides in astronomy. He explained the mechanics behind planetary motion and eclipses, and notably proposed a heliocentric view in which the Earth rotates on its axis and revolves around the Sun. The Āryabhatī**y**a contains detailed astronomical tables, including planetary data, which enabled more accurate forecasting and celestial observations by future astronomers. Aryabhata's contributions left a lasting legacy in both mathematics and astronomy, profoundly shaping their development in India and influencing scholars for generations. His innovative ideas crossed cultural and geographic boundaries, reaching the Islamic world and eventually medieval Europe, where they played an important role in the global advancement of mathematical knowledge. His work served as a foundation for future research and helped bridge intellectual traditions across civilizations.

2.4 ARYABHATIYA CONTRIBUTION IN MATHEMATICS

Aryabhata introduced the concept of **zero,** place value system, and worked extensively on **algebra, trigonometry**, and **spherical geometry.** He approximated the value of **pi (π)** to an extraordinary degree of accuracy and explained that the Earth rotates on its axis, which was a revolutionary idea for his time. His works had a profound influence not only in India but also on Islamic and European mathematics.

(i) PLACE VALUE SYSTEM: He used a positional decimal system and even hinted at the use of zero. He used a system of representing numbers where 33 consonants of the Indian alphabetical system were used to denote numbers from 1 to 100, and numbers up to 10^18 could be represented with alphabetical notation. However, Aryabhata did not use the Brahmi numerals. Continuing the Sanskrit tradition from Vedic times, he used letters of the alphabet to denote numbers, expressing quantities, such as the table of sines in a mnemonic form.

(ii) ALGEBRA AND TRIGNOMETRY: Aryabhata developed techniques to solve both linear and quadratic equations and presented formulas for calculating the sums of arithmetic and geometric series, showcasing his deep grasp of algebraic principles. He presented detailed sine tables in his text and his trigonometric formulations paved the way for later advancements in the field.Aryabhata's legacy continues to inspire mathematicians and scientists around the world, and he is remembered as a pioneer of mathematical is thought in ancient India.

(iii) CONTIBUTION TO ASTRONOMY: Explained the eclipse**s** scientifically: lunar eclipses occur due to Earth's shadow and not because of mythical causes.Calculated the sidereal rotation (the time taken for Earth to rotate relative to fixed stars) accurately.Estimated the length of the solar year as 365 days, 6 hours, 12 minutes, and 30 seconds—only a few minutes off the modern value.

Aryabhata's contributions to mathematics and astronomy have had a profound influence on subsequent generations of mathematicians and scientist. His work was translated into Arabic and played a crucial role in the development of mathematical astronomy in the Islamic world. His ideas continue to be foundation in various fields of mathematics and science today.

2.5 BRAHMAGUPTA LIFE

Brahmagupta was a renowned Indian mathematician and astronomer born in 598 CE in Bhinmal, Rajasthan, India. He was a renowned Indian mathematician and astronomer of the 7th century CE, whose ground-breaking contributions laid the foundation for several key concepts in mathematics. Brahmagupta served as the head of the astronomical observatory at Ujjain, which was a major center for learning and scientific research at the time. His most famous work, the *Brāhmasphuṭasiddhānta* (628 CE), contains pioneering ideas in arithmetic, algebra, and astronomy. He was the first to define zero as a number and establish rules for arithmetic operations involving zero and negative numbers. Brahmagupta also devised methods for solving quadratic equations and provided rules for dealing with positive and negative numbers, paving the way for modern algebra. His writings demonstrate a deep understanding of mathematics and its applications in astronomy, influencing both Indian and Islamic scholars for centuries to come.

2.6 BRAHMAGUPTA CONTRIBUTION IN MATHEMATICS

Brahmagupta made significant contributions to mathematics and astronomy that had a lasting impact on both Indian and global scientific traditions. Some of his most important contributions include:

(i) INTRODUCTION OF ZERO AS A NUMBER: Brahmagupta was the first to treat zero as a number in its own right and developed rules for arithmetic operations involving zero (such as addition, subtraction, and multiplication), which laid the foundation for modern number systems.

(ii) RULES FOR NEGATIVE NUMBERS: He established consistent rules for operating with positive and negative numbers, including how to subtract a larger number from a smaller one — a concept not clearly defined before his time.

(iii) ALGEBRAIC INNOVATIONS: Brahmagupta made major advances in algebra. He provided general solutions for quadratic equations, including cases involving negative discriminants, and developed methods for solving indeterminate equations.

(iv) ARITHMETIC AND GEOMETRY: He discussed operations on fractions, square roots, and cube roots. In geometry, he gave formulas for the area of cyclic quadrilaterals (now known as **Brahmagupta's formula**) and contributed to understanding the properties of triangles and circles.

(v) ASTRONOMY: He discussed operations on fractions, square roots, and cube roots. In geometry, he gave formulas for the area of cyclic quadrilaterals (now known as **Brahmagupta's formula**) and contributed to understanding the properties of triangles and circles.

(vi) MATHEMATICAL ASTRONOMY: He emphasized the application of mathematics to astronomical problems and encouraged observational validation of theoretical predictions.

Brahmagupta's works were translated into Arabic and influenced Islamic and later European mathematics, making him one of the most important figures in the history of mathematical science.

2.7 BHASKARA I LIFE AND WORK

Bhaskara I was born in the regions of Saurashtra or Asmaka around 600 CE. He was one of the earliest and most respected Indian mathematicians and astronomers of the **7th century CE.** These areas were important cultural and scholarly centers during that period and are now part of present-day Gujarat and Maharashtra in western India. His birthplace in these historically significant regions suggests that he may have been influenced by vibrant traditions of mathematics and astronomy that flourished there. He is known for his important **commentaries on the works of Aryabhata,** especially the **Aryabhatiya,** and for developing some of his own original ideas in **mathematics and astronomy.**

https://www.cuemath.com/learn/bhaskara Fig 2.1

The information about the life of Bhaskara I is very limited, and there is no historical record of the exact date or year of his death. Historians estimate that he lived between 600 and 680 AD. Therefore, they believed that Bhaskara I must have died towards the end of the 7th century (around 680 AD).

BHASKARA I WORK

(i) ARITHMETIC AND NUMBER SYSTEM: Bhaskara I's mathematical insights closely align with the foundational principles of Vedic Mathematics. He effectively explained and applied the **place-value system,** which is central to many Vedic techniques. This system, inherited from Aryabhata's work, enabled the easy handling of large numbers and is the basis for **sutras l**ike *Nikhilam* and *EkadhikenaPurvena*. These sutras simplify multiplication, division, and squaring by using the base value method (e.g., 10, 100, 1000).

Bhaskara I also recognized the role of **zero as a placeholder**, even before its symbol became standard. This understanding laid the groundwork for the **positional number system** that Vedic Mathematics depends upon.

Example 3: Square a number ending in 5 (Vedic Sutra: EkadhikenaPurvena)

Solution: Let the number is65^2

Step 1: Take the first digit(s): 6
Step 2: Multiply $6 \times (6 + 1) = 6 \times 7 = 42$
Step 3: Always add **25**at the end
Answer: 65^2 = **4225**

Explanation of the example

In this example we use place value: Here 6 is in the **tens** place, so 60^2 is approx. 3600. Then $5^2 = 25$ is fixed. The extra comes from how 60 × 70 = 4200 → +25 = **4225**

Furthermore, Bhaskara I's use of **logical reasoning and example-based teaching** resonates strongly with the mental strategies promoted in Vedic Mathematics. His step-by-step approach made complex calculations simpler and encouraged intuitive thinking—core to the Vedic tradition of performing fast, accurate mental calculations using sutras. In this way, Bhaskara I's classical methods can be seen as early expressions of the powerful mental techniques later formalized in Vedic Mathematics.

(ii) IN ALGEBRA: The development of algebra in ancient India was significantly influenced by thinkers like Aryabhata and Bhaskara, who laid foundational principles in symbolic and verbal reasoning. Aryabhata interpreted and expanded algebraic rules, especially in the realm of indeterminate equations and arithmetic series, using pattern recognition and general formulae to uncover and explain mathematical relationships. Later, Vedic mathematics introduced algebraic sutras such as **śūnyaṁ sāmya-samuccaye** ("If the sum is the same, the sum is zero") and **parāvartya yojayet** ("Transpose and adjust"), which encapsulate powerful algebraic ideas in compact, intuitive forms.

(iii)APPROXIMATION OF SINE FUNCTION: In Vedic Mathematics, trigonometric functions like sine are not typically approximated in the way modern calculus does with Taylor series. However, Vedic Mathematics does offer some clever approximations and simplified methods that are easy to compute mentally or with minimal tools. One such example is the **Bhāskarācārya approximation for sine**, which predates modern calculus and can be considered part of ancient Indian mathematics tradition—closely related to what we now call Vedic or traditional Indian mathematics.

Example 1: Approximation of Sine Function (Bhāskara's Formula)

He gave a very accurate formula to calculate sine values, which was impressive for his time:

Bhāskara II (12th century) gave a remarkably accurate approximation for sine:

$$\text{Sin}\,\theta \approx \frac{4\,\theta\,(180-\theta)}{40500-\theta\,(180-\theta)}$$

Where θ is in degree and 0∘≤θ≤180∘0

Example 2: Let us compute $\sin(30)^0$

$$\sin(30)^0 \approx \frac{4.90.(180-90)}{40500} = \frac{4.90.90}{40500} = \frac{32400}{40500} = 0.8$$

Here it’s **off by a bit** (true value is 1), but it's still a fast mental approximation.

IT is Easy to compute using mental multiplication and division. No need for calculator, tables, or series expansion. Captures the general shape of the sine curve

(iv) USE OF PLACE-VALUE SYSTEM AND ZERO:

Bhaskara I effectively used the place-value system, an essential concept in Vedic-style calculations. His understanding of zero (as a placeholder) was implicit, aligning with mental techniques used in Vedic Mathematics for rapid arithmetic.

Example 1: Multiply 23 × 11 using place-value logic (Vedic Method)

Step 1: Separate the digits of 23 → 2 _ 3
Step 2: Add the two digits: 2 + 3 = 5
Step 3: Place the sum in between → Answer: 253

So, 23 × 11 = 253

Here we explained how does this relate to the Place-Value System**?**

The number 23 = (2 × 10) + (3 × 1). In this method We are using the **place values** of digits (tens and units). The middle digit (5) represents the sum of digits at different place values.

This method is based on the sutra:
“Antyayoreva” – meaning *only the ends* (used in special multiplication cases).

Example 2: Multiply 45 × 11

Step-by-Step (Vedic Method using Place Value):

Step1: Split the digits: 4 _ 5
Step 2: Add the two digits: 4 + 5 = 9
Step 3: Place the result in between**: 495**

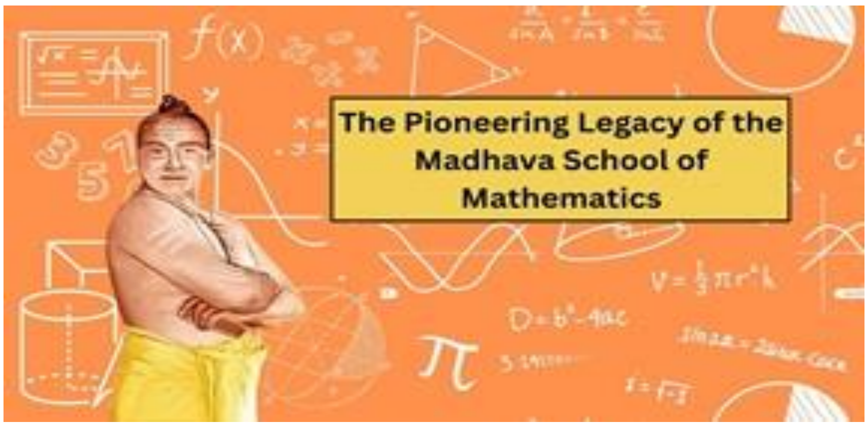
So, **45 × 11 = 495**

Explanation:
Here, 4 is in the **tens**place and 5 in the **units.** The sum of the place-value digits (4 + 5) gives the middle number. This trick works due to the distributive property of numbers and place values in the decimal system.

His work laid the **foundation for later Indian scholars**like Brahmagupta and Bhāskara II. He demonstrated**scientific thinking and precision**in both **mathematics and astronomy,** long before these ideas were developed in Europe.

2.8 MADHVA'S LIFE AND WORK

Madhava was born around 1340 AD in a small village called Sangamgram, hence he is also called Sangamgram ka Madhava. Sangam gram is believed to be near the present-day town of Irinjalakuda in Kerala, South India. He belonged to a Brahmin family, and like many scholars of his time, he was deeply involved in both mathematics and astronomy. His education likely included the study of the Vedas, Sanskrit, and Jyotisha (astronomy), which was closely connected to mathematical practice in ancient India. Madhava of Sangamagrama (c. 1340 – 1425 CE) is a towering figure in the history of Indian mathematics and astronomy. Though not directly a part of the ancient Vedic period, Madhava's work reflects the rich tradition of mathematical inquiry that evolved in India and later influenced the so-called Vedic Mathematics revival. He is widely regarded as the founder of the **Kerala School of Mathematics**. He taught many brilliant learners, and his teachings were passed down through generations. Scholars like **Paramesvara, Nilakantha Somayaji,** and others preserved and expanded on his work, ensuring that his contributions were not lost.



Pic Credit: The Verandah Club Fig 2.2.

Although not much is recorded about his personality or family life, Madhava is remembered as a **brilliant thinker, humble teacher,** and **dedicated scholar**. He lived a simple life focused on learning and teaching. Unlike many Western mathematicians whose works were widely published, Madhava's ideas were transmitted through **manuscripts and oral tradition**, which is why many of his original texts are now lost or known only through quotations by later scholars.

Madhava is believed to have died around **1425 CE**, but the exact date and circumstances of his death are unknown. His intellectual legacy lived on through his learners and the Kerala School, which continued to influence Indian mathematics for over two centuries.

MADHVA'S WORK

Madhava of Sangamagrama stands as one of the greatest Indian mathematicians, whose discoveries in infinite series and early calculus mark a golden era in Indian science. His work, rooted in traditional Sanskrit scholarship and original insight, continues to inspire mathematicians and historians alike. Madhva of Sangamagrama (c.1340-1425) made great contributions to the history of Indian mathematics, but he did not produce anything called "Vedic Mathematics." His contributions were part of the ancient Indian Mathematical tradition, but they are not directly connected to Bharati Krishna Tirtha;s " Vedic Mathematics". Yet Madhava work was invaluable to the development of Indian mathematics: Here are his main contribution, which are considered part of the great tradition of a ancient Indian Mathematics.

(i) INFINITE SERIES: Madhava developed infinite series for sine,cosine, arctangent etc. this was the foundation of modern calculus.

Example: Madhava series for pi

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5}$$

If we add the first few terms of this series, its value gradually approaches $\frac{\pi}{4}$.

Till first term: 1 = 1.00000000

Till second term:$1- \frac{1}{3}$ = 0.66666667

Till three terms: 0.66666667 +$\frac{1}{3}$ = 0.8666667

Till four terms: 0.8666667-$\frac{1}{7}$ = 0.72380952

In this way, the more terms you add, the closer the value gets to $\frac{\pi}{4}$.

$$\pi \approx 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right)$$

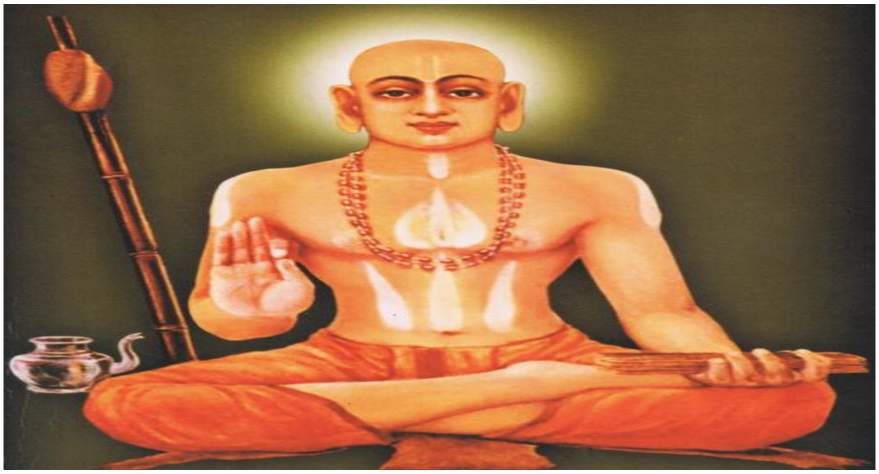
But Madhava realized the problem of slow convergence of this series and added a correction term, which allowed him to obtain the value of π correct to 11 decimal places:

$\pi \approx 3.14159265359$

This was the most accurate value in the world in the 14th century, such an accurate value of pi was unimaginable in Europe at that time.

2.9 MADHVACHARYA'S LIFE AND WORK

Madhvacharya was born in the 13th century on the western coast of India, in a Tulu-speaking Brahmin family at Pajaka, a place near Udupi in the state of Karnataka, to Madhyageha Bhatta and Vedavati. From a young age, he demonstrated extraordinary intellectual and spiritual abilities, and in his adolescence, he became a monk and joined the Brahma Sampradaya under the Ekadandi order with his guru Achyutapreksha.



https://wikimili.com/en/Madhvacharya *Fig:2.3*

He is primarily known for his philosophical contributions, but his works also touch upon various aspects of Vedic knowledge, including mathematics. He stressed devotion (*bhakti*), knowledge, and the pursuit of truth, leaving a lasting impact on Hinduism. He established the Udupi Sri Krishna Matha and eight monasteries (*ashtamathas*) to serve the deity "Udupi Krishna." He wrote 37 works in Sanskrit, including the *Anuvyakhyana* and the *Sarva-Darshana-Sangraha*, a compendium of various philosophical systems. He influenced Vaishnavism and the Bhakti movement, offering comfort and guidance to countless individuals on their spiritual journeys. Madhvacharya's teachings continue to inspire both seekers and scholars alike, providing a profound understanding of the divine reality and the relationship between the individual and God.

Although there is not enough information about Madhvacharya's direct contributions to Vedic mathematics, his philosophical works and legacy have continued to influence Indian thought and intellectual traditions. Madhvacharya is believed to have lived 79 years, from 1238 to 1317CE. According to traditional accounts, he passed away in 1317 CE, leaving behind a legacy of philosophical and theological contributions that continue to influence Hindu thought and spirtualiti.

MADHVACHARYA'S WORK

When people hear **"Vedic Mathematics,"** they sometimes think it refers to the *actual mathematical works* of ancient Vedic scholars like Madhvacharya. But the system popularly called "Vedic Mathematics" today mostly comes from the 20th-century book *Vedic Mathematics* (1965) by Bharati Krishna Tirthaji, who claimed to have reconstructed ancient techniques from the Vedas — a claim that remains historically unsubstantiated. He was a prominent philosopher and theologian, founder of the Dvaita (dualism) school of Vedanta. His main works were on Vedantic philosophy, commentaries on the Brahmasutras, Upanishads, and the Bhagavad Gita — not mathematics.

2.10 BHASKARAII LIFE AND WORK

Bhaskara II (1114–1185 CE) was a prominent 12th-century Indian mathematician and astronomer who made significant contributions to both fields. He also known as Bhaskaracharya, He is renowned for his work on arithmetic, algebra, geometry, and astronomy, particularly his systematic use of the decimal number system. He also did important work in astronomy, including the study of planetary motions and other celestial phenomena. Bhaskaracharya's work helped give new direction to Indian mathematics and astronomy.

BHASKARA II WORK

Bhāskara's most famous contributions are collected in his four-part mathematical treatise called the "Siddhānta Shiromani" ("Crown of Treatises"). Each of these books shows Bhāskara's deep understanding and creative thinking in mathematics and astronomy. Here is some detailed knowledge given below.

LĪLĀVATĪ (ARITHMETIC); Līlāvatī is the first part of Bhāskara II's great treatise Siddhānta Shiromani, written in 1150 CE. It is a beautiful and poetic work on arithmetic and elementary mathematics. behind the name There is a famous story. According to legend: Bhāskara's daughter Līlāvatī was destined never to marry. To help her avoid this fate, Bhāskara tried to find a perfect, auspicious time for her wedding using a water-clock. A pearl from Līlāvatī's dress fell into the clock, disturbing the calculation. The wedding failed, and Līlāvatī remained unmarried. To comfort her, Bhāskara named his arithmetic book after her, so her name would be remembered forever. Līlāvatī became a standard textbook in India for several centuries. It was translated into Persian, Arabic, and other languages and studied by mathematicians worldwide. Many math teachers still refer to its poetic charm and clarity in problem-solving. In modern India, the "Līlāvatī Award" is given by the Indian National Science Academy to women for outstanding contributions to mathematics.

Bījagaṇita (Algebra): Bījagaṇita is the second part of Bhāskara II's famous treatise Siddhānta Shiromani (written in 1150 CE) and is entirely devoted to algebra — a branch of mathematics dealing with symbols, variables, Solution to linear equation and Quadratic equation. Bhāskara gave **rules for finding roots**and even acknowledged **negative roots**. They have also operations involving non perfect square roots like $\sqrt{2}$, $\sqrt{5}$. He showed how to simplfy and **use surds in equations**.It is one of the**earliest systematic treatiseson algebra**in the world.

Chakravāla Method: This method is an advanced **cyclic method** developed by Bhāskara for solving **indeterminate equations**. This method was **highly accurate** and used **trial and error with intelligent steps.**It is considered one of **India's greatest contributions to algebra.**

Grahagaṇita and Golādhyāya (Astronomy): Grahagaṇita Described the rotational movement of Earth nearly 500 years before Galileo. And accurately calculated planetary motions, eclipses, and the length of a year. Explained how the moon and planets do not have their own light, but reflect sunlight.

2.11 VARAHAMIHIRA LIFE AND WORK

Varāhamihira was born into a Brahmin familyAround 505 CE, likely in the region of Avanti (modern-day Malwa, central India). His father, Adityadasa, was also an astrologer and introduced him to astronomy and astrology. Inspired by Greek and Roman astronomical ideas, he combined them with Indian traditions to create more accurate models. He spent much of his life in Ujjain, which was one of India's greatest centers for astronomy and mathematics.He became a renowned scholar in King Yashodharman's court and was recognized as one of the Navaratnas (Nine Gems) of the king's assembly.His works show he was familiar with the suryasiddhanta and also knowledge from outside India (e.g., Greek and Roman astronomy), which he integrated skillfully. Varāhamihira was not just an astronomer but a true polymath: he wrote on meteorology, hydrology, geology, botany, architecture, and omens.His keen observations of nature and emphasis on practical applications of astronomy made his works unique.He authored texts like Pancha-Siddhāntikā, which preserved knowledge of five astronomical traditions, and BṛhatSaṃhitā, a massive encyclopedia of the sciences of his time.He made significant improvements in the calculation of eclipses, planetary positions,and trigonometric functions.Varāhamihira's texts were studied for centuries across India.He played a vital role in shaping classical Indian astronomy and astrology.His clear explanations and extensive observations made his work influential not only in India but also in neighboring regions.

VARAHAMIHIRA WORK

The term **“Vedic Mathematics”** generally refers to the system popularized by Bharati Krishna Tirthaji in the 20th century, which claims to be based on “16 sutras from the Vedas.” However, these sutras are not found in any ancient Vedic texts, and the methods described are Tirthaji’s own reconstructions or reinterpretations. Varāhamihira lived in the **6th century CE**, which was many centuries after the Vedic period (1500–500 BCE).

Varāhamihira’s writings, such as the *Pancha-Siddhāntikā* and *BṛhatSaṃhitā*, were based on the **Vedāṅga Jyotisha tradition**, which is an auxiliary discipline of the Vedas related to astronomy and calendars. He preserved, compared, and refined knowledge from the **SūryaSiddhānta** (which itself is linked to older Jyotisha) as well as other astronomical traditions like the RomakaSiddhānta. He focused on **astronomical calculations**, including Computing planetary positions and conjunctions, predicting eclipses, Calculating solstices and equinoxes. He compiled **trigonometric tables using sine functions,** which were important for both astronomy and what we now call spherical geometry. His works also included practical applications such as: Weather prediction, Signs of earthquakes, Architecture and city planning, Omens and astrology.

CHECK YOUR PROGRESS

TRUE OR FALSE

1. The name of Bhāskara's famous mathematical treatise is Siddhānta Shiromani.
2. Bhāskara introduced a method to solve Pell’s Equation, known as:Chakravala Method
3. Bhāskara II’s Līlāvatī is primarily a text on:
4. Arithmetic. Madhava’s work laid the foundations of which later mathematical field is calculus.

MULTIPLE CHOICE QUESTIONS

1. **Aryabhata was born in which year?**
   a) 476 CE
   b) 598 CE
   c) 100 CE
   d) 1200 CE

2. **Which of the following is a famous work of Aryabhata?**
   a) Brāhmasphuṭasiddhānta
   b) Āryabhaṭīya
   c) SūryaSiddhānta
   d) Lilavati

3. **Aryabhata proposed that the Earth:**
   a) Is flat

b) Is supported by elephants
c) Rotates on its axis
d) Does not move

4. **Aryabhata's place-value system was significant because:**
a) It ignored the concept of zero
b) It used Roman numerals
c) It allowed calculation of large numbers efficiently
d) It only worked with fractions

5. **Brahmagupta was born in which year?**
a) 476 CE
b) 1000 CE
c) 598 CE
d) 820 CE

6. **Brahmagupta's most famous book is:**
a) Āryabhaṭīya
b) Brāhmasphuṭasiddhānta
c) Lilavati
d) Bijaganita

7. **Brahmagupta is credited with formalizing which mathematical concept?**
a) Infinity
b) Zero as a number
c) Trigonometry
d) Logarithms

8. **Brahmagupta's formula is used to calculate the area of a:**
a) Triangle
b) Circle
c) Cyclic quadrilateral
d) Parallelogram

9. **Brahmagupta gave rules for operating with:**
a) Only whole numbers
b) Only positive numbers
c) Negative numbers and zero
d) Complex numbers

10. **Which book by Bhāskara II is written in a poetic style and focuses on arithmetic?**
a) Bījagaṇita
b) Golādhyāya
c) Līlāvatī
d) Grahagaṇita

11. Madhava of Sangamagrama is best known for his early work on which mathematical concept?

a) Place value system
b) Infinite series
c) Decimal fractions
d) Geometry of circles

12. Varāhamihira’s famous encyclopedia that covers astrology, weather, architecture, and omens is called:

a) Līlāvatī
b) Pancha-Siddhāntikā
c) BṛhatSaṃhitā
d) Khandakhadyaka

2.12 SUMMARY

In this chapter, the life of the great mathematicians of ancient India, Aryabhata and Brahmagupta and their important mathematical inventions have been described. Aryabhata studied the decimal system, number calculation, trigonometry and planetary movements. Brahmagupta used zero clearly and wrote an important book called Brahmasphuṭasiddhāntaḥ. The works of these mathematicians not only developed Indian mathematics, but their influence was also seen in Islamic and European mathematics. Their inventions established fundamental foundations in many fields of mathematics, astronomy and science.

Bhāskara II: Bhāskara II, also known as Bhaskaracharya, was one of the greatest mathematicians and astronomers of ancient India. Born in 1114 CE in Bijapur, he made extraordinary contributions to arithmetic, algebra, geometry, and astronomy. His famous work *Siddhānta Shiromani* is divided into four parts:

Bhāskara I: Bhāskara I was an important Indian mathematician and astronomer who lived in the 7th century CE. He is best known for writing commentaries on Aryabhata's work, especially the famous book Aryabhatiya. His explanations helped make Aryabhata's complex ideas easier to understand. He is especially famous for giving a very accurate formula for calculating sine values, which showed his deep understanding of mathematics.He was one of the earliest scholars to use the decimal number system and the concept of zero, which were revolutionary ideas at that time. Bhāskara I’s writings played a big role in preserving and spreading Aryabhata’s ideas to future generations. His work showed logical thinking, clarity, and a love for knowledge, and he influenced many Indian mathematicians who came after him.

2.13 GLOSSARY

i. **Aryabhata:** Ancient Indian mathematician and astronomer, born in 476 CE. Author of *Āryabhaṭīya.*
ii. **Āryabhaṭīya**: A major work by Aryabhata covering mathematics and astronomy. Written in verse form.
iii. **Brahmagupta:**7th-century Indian mathematician and astronomer, born in 598 CE in Bhillamala (modern Bhinmal).
iv. **Brāhmasphuṭasiddhānta**:Brahmagupta's most famous work, written in 628 CE, covering mathematics and astronomy.
v. **Zero (as a number):** Brahmagupta was the first to define zero as a number and set rules for operations with it.
vi. **Bhāskara II / Bhaskaracharya:** A famous 12th-century Indian mathematician and astronomer known for his works like *Līlāvatī* and *Bījagaṇita*.
vii. **Līlāvatī:** A book on arithmetic written by Bhāskara II, named after his daughter, containing poetic math problems.
viii. **Siddhānta Shiromani:** A major work by Bhāskara II, meaning "Crown of Treatises", which includes four parts on arithmetic, algebra, astronomy, and spherical geometry.
ix. **Grahagaṇita:** The part of Bhāskara's work those deals with planetary mathematics and astronomy.
x. **Navaratnas** : Literally "Nine Gems"; a group of nine distinguished scholars in an Indian royal court, such as those in the court of King Yashodharman.
xi. **Varāhamihira :** A 6th-century Indian astronomer and polymath, famous for works like *Pancha-Siddhāntikā* and *BṛhatSaṃhitā*.

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4. *Astāngahrdaya*,Vol.I, *Sūtrasthāna* and *Śarīrasthāna*, Translated by K. R. Srikantha Murthy, Vol. I, Krishnadas Academy, Varanasi, 1991.
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2.15 SUGGESTED READING

1. **"Vedic Mathematics" by Bharati Krishna Tirthaji:** The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras.Covers mental math techniques for arithmetic, algebra, and geometry
2. **"Vedic Mathematics Made Easy" by Dhaval Bathia:** A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.
3. **"The Power of Vedic Maths" by Atul Gupta:** Focuses on speed and accuracy.Ideal for practical application in exams like CAT, GRE, and banking tests.
4. **"Mathematics in India" by Kim Plofker**: A scholarly account of the historical development of mathematics in India.Provides context on how Vedic methods evolved over time.
5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

2.16 TERMINAL QUESTIONS

1. Describe the early life and major contributions of Aryabhata in the fields of mathematics and astronomy.
   ………………………………………………………………………………………..
2. Explain Aryabhata's astronomical concepts and theories. How did they differ from the prevailing beliefs of his time?
   ……………………………………………………………………………………….
3. Write a detailed account of Brahmagupta's life, his key mathematical contributions, and his influence on later mathematicians.
   ………………………………………………………………………………………………..
4. Discuss Brahmagupta's contributions to number systems, particularly his treatment of zero and negative numbers.
   …………………………………………………………………………………………………
5. Evaluate Brahmagupta's contributions to geometry and algebra. Explain the significance of his formula for the area of a cyclic quadrilateral.
   ………………………………………………………………………………………………….

2.17 ANSWERS

CYQ1. TRUE	CYQ2. TRUE	CYQ3. TRUE	CYQ4. TRUE
MCQ1. A	MCQ2. B	MCQ3. C	MCQ4. C
MCQ5. C	MCQ6. B	MCQ7. B	MCQ8. C
MCQ9. C	MCQ10.C	MCQ11. B	MCQ12. C

UNIT 3- SRINIVASA RAMANUJAN

CONTENTS:

3.1 INTRODUCTION

Just getting to know the life and work of the amazing mathematician Srinivasa Ramanujan is enough to make anyone marvel at the potential of human genius. Ramanujan's mathematical work is not considered simple. Some Mathematicians consider his formulas to be extremely complex. Srinivasa Ramanujan Aiyangar, despite no formal training in pure mathematics, still made a substantial contribution to it. His mathematical findings were too novel, unfamiliar and unusual so much that. Previous unit explored the lives and works of four such luminaries: Bhāskara I Bhāskara II, Varāhamihira, and Madhava of Sangamagrama. Now In the beginning of the present unit, circumstances of Ramanujan's life have been highlighted and in the other part, some examples of mathematics have been given which are necessary to be mentioned. I am confident that after reading this unit dear learners you will not only get acquainted with the works of Srinivasa Ramanujan but will also get inspired.

3.2 OBJECTIVES

After studying this unit learner will be able to:

1. Explained about the life and education of Srinivasa Ramanujan.
2. Describe the work of Ramanujan in the field of Mathematics.

3. Familiar with the thoughts of different mathematicians regarding Ramanujan life and work.

3.3 RAMANUJAN LIFE AND FAMILY

Ramanujan was born on **22 December 1887**, into a Tamil Brahmin Iyengar family in Erode, in present-day Tamil Nadu His father, Kuppuswamy Srinivasa Iyengar, originally from Thanjavur district, worked as a clerk in a sari shop. His mother, Komalatammal, was a housewife and sang at a local temple. They lived in a small traditional home on Sarangapani Sannidhi Street in the town of Kumbakonam. The family home is now a museum.

On 1 October 1892, Ramanujan was enrolled at the local school. After his maternal grandfather lost his job as a court official in Kanchipuram, Ramanujan and his mother moved back to Kumbakonam, and he was enrolled in Kangayan Primary School. When his paternal grandfather died, he was sent back to his maternal grandparents, then living in Madras.

He did not like school in Madras, and tried to avoid attending. His family enlisted a local constable to make sure he attended school. Within six months, Ramanujan was back in Kumbakonam. Since Ramanujan's father was at work most of the day, his mother took care of the boy, and they had a close relationship.

A child prodigy by age 11, he had exhausted the mathematical knowledge of two college learners who were lodgers at his home. He was later lent a book written by S. L. Loney on advanced trigonometry. He mastered this by the age of 13 while discovering sophisticated theorems on his own. By 14, he received merit certificates and academic awards that continued throughout his school career, and he assisted the school in the logistics of assigning its 1,200 learners (each with differing needs) to its approximately 35 teachers.

In 14 July 1909, Ramanujan married Janaki a girl his mother had selected for him a year earlier and who was ten years old when they married. In 1912, Janaki and Ramanujan's mother joined Ramanujan in Madras. To make money, he tutored learners at Presidency College who were preparing for their Fellow of Arts exam. In May 1913, upon securing a research position at Madras University, Ramanujan moved with his family to Triplicane.

In the spring of 1913, Narayana Iyer, Ramachandra Rao and E. W. Middlemast tried to present Ramanujan's work to British mathematicians. M. J. M. Hill of University College London commented that Ramanujan's papers were riddled with holes.He said that although Ramanujan had "a taste for mathematics, and some ability", he lacked the necessary educational background and foundation to be accepted by mathematicians. Although Hill did not offer to take Ramanujan on as a learner, he gave thorough and serious professional advice on his work. With the help of friends, Ramanujan drafted letters to leading mathematicians at Cambridge University.The first two professors, H. F. Baker and E. W. Hobson returned Ramanujan's papers without comment. On 16 January 1913, Ramanujan wrote to G. H. Hardy whom he knew from studying *Orders of Infinity* (1910). Coming from an unknown mathematician, the nine pages of mathematics made Hardy initially view Ramanujan's manuscripts as a possible fraud. Hardy recognized some of Ramanujan's formulae but others "seemed scarcely possible to believe".On 17 March 1914, Ramanujan travelled to England by ship, leaving his wife to stay with his parents in India.

Fig.2.3.1 (Ref: https://hi.wikipedia.org/wiki)

On 6 December 1917, Ramanujan was elected to the London Mathematical Society. On 2 May 1918, he was elected a Fellow of the Royal Society the second Indian admitted, after ArdaseerCursetjee in 1841. In 1919, Ramanujan returned to Kumbakonam, Madras Presidency, **where he died in 1920 aged 32.** After his death, his brother Tirunarayanan compiled Ramanujan's remaining handwritten notes, consisting of formulae on singular moduli, hypergeometric series and continued fractions.In 1976, George Andrews was examining **G. N. Watson's papers at Trinity College Library** and found a sheaf of 138 pages in Ramanujan's handwriting. he notebook is considered a major discovery, containing mathematical material that emanated from the last year of Ramanujan's life, shortly before his death.

CHECK YOUR PROGRESS

CHQ1. When was Srinivasa Ramanujan born?

i. 22 December 1887
ii. 7 April 1889
iii. 30 June 1887
iv. 3 March 1885

CHQ2. In which town in South India was Ramanujan born?

i. Erode
ii. Chennai
iii. Bangalore
iv. Hyderabad

3.4 RAMANUJAN MATHEMATICAL CONTRIBUTION

3.4.1 NUMBER THEORY AND PARTITION FUNCTIONS

- **Partition of Numbers**: Ramanujan developed a theory for partition functions, which counts the ways a positive integer can be expressed as a sum of positive integers. His insights into these functions and their properties significantly advanced Combinatorics and Number Theory.
- **Partition of a integer:** In simple language, partition of an integer is the number of ways the integer can be expressed as sum of positive integers. For example, consider n=5. Then n=5 can be expressed as a sum of positive numbers in following ways:

$$\begin{aligned} 5 &= 5 \\ &= 4+1 \\ &= 3+2 \\ &= 3+1+1 \\ &= 2+1+1+1 \\ &= 2+2+1 \\ &= 1+1+1+1+1 \end{aligned}$$

The values of P(n) are 1,2,3.5,7,11,… for n=1,2,3,4,5,6… . As the value of n increase, the value of p(n) increase exponentially. For instance, the value of P(10) is 47 while the value of P(100) is 190569292. Instead of calculating the value of P(n), many mathematicians tried to formulate this partition function P(n). Srinivasa Ramanujan was one of them.

- **Contribution of Srinivasa Ramanujan to the partition function:**

  It is very difficult to calculate partition of a large positive number. The contribution of Srinivasa Ramanujan to calculate the value of partition of large number is very significant. Ramanujan, in collaboration with G. H. Hardy developed a formula to find the value of partition of large number. His formula is as follows:

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{2\pi\sqrt{\frac{n}{6}}}$$

The above formula doesn't provide the exact value but a very close approximation for large value of n. Ramanujan also developed various properties related to the partition function. Some of them are as below:

$$P(5n+4) \equiv 0 \ mod\ 5.$$
$$P(7n+5) \equiv 0 \text{ mod } 7;$$
$$P(11n+6) \equiv 0 \ mod\ 11.$$

- **Prime Numbers**: He identified properties of special numbers like primes and patterns within number sets, contributing to the study of number theory. Infinite Series & Mathematical Analysis.

- **Divergent Series**: Ramanujan introduced the concept of Ramanujan Summation, a new method for summing series that otherwise diverge (go on infinitely).

- **Pi(π) :** He discovered several remarkable, finite formulas for computing the digits of Pi.

Pi is a mathematical constant which was first calculated by Archimedes. He found its value to be 3.14159265358979… . In Mathematical language pi is defined to be the ratio between circumference of a circle and its diameter. In the history of mathematics, it has been always a challenge for mathematicians to calculate the correct approximation of pi.

In the early age, Srinivasa Ramanujan had remarkable talent for mathematics. Though he had many problems in his early life but his interest towards mathematics remained unflinching. He slowly became popular for his extraordinary talent for mathematics despite lack of a university education. He wrote letters mentioning his results to many prominent mathematicians of that time trying to interest them in his results. His letters were ignored at first until the potentiality of his results were recognized by G.H. Hardy.

In the letter to Hardy, Ramanujan mentioned many infinite series, the famous one was the following series for pi:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!\ (1103 + 26390n)}{(n!)^4 396^{4n}}$$

- **Special Functions**: He provided formulas for functions like hypergeometri series and elliptic integrals that were not widely known, which later proved influential.

3.4.2 CONTINUED FRACTIONS AND MODULAR FORMS

- **Continued Fractions**: Ramanujan studied continued fractions, leading to novel expressions for numbers like Pi and connections between these fractions and modular forms.
- **Modular Forms**: His research revealed profound relationships within modular functions and their applications to other mathematical areas, laying groundwork for future developments in the field.
- **Mock Theta Functions**: In his final year, Ramanujan introduced mock theta functions, a significant development that has found applications in areas such as string theory and black hole physics.
- **Rogers-Ramanujan Identities**: He also discovered the famous Rogers-Ramanujan identities, which have since found surprising connections to statistical mechanics and representation theory.

3.4.3 1729 RAMANUJAN NUMBER

1729 is known as the Ramanujan number or Hardy–Ramanujan number. It is the smallest number that can be expressed as the sum of two positive integer cubes in two different ways:

$$1^3 + 12^3 = 1729 = 9^3 + 10^3.$$

1729 is the smallest number for which this property holds true, earning it the title of the smallest taxicab number. G.H. Hardy visited Ramanujan, who was ill in the hospital in London.Hardy mentioned that the number on his taxi, 1729, seemed like a rather ordinary number. Ramanujan immediately responded, "No, it is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways".

3.5 HARDY WROTE OF RAMANUJAN

"The limitations of his knowledge were as startling as its profundity. Here was a man who could work out modular equations and theorems... to orders unheard of, whose mastery of continued fractions was... beyond that of any mathematician in the world, who had found for himself the functional equation of the Zeta function and the dominant terms of many of the most famous problems in the analytic theory of numbers; and yet he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was..."

"In his favorites topics, like infinite series and continued fractions, he had no equal this century. His insight into algebraic formulae, often (and unusually) brought about by considering numerical examples, was truly amazing. But in analytic number theory, a subject he is often associated with, I do not believe he actually knew that much. He certainly contributed little of significance that was not known already. And in a subject that relied so much on proof, a subject where intuition had a bad habit of coming unstuck, he produced much that was false." "I remember once going to see [Ramanujan] when he was lying ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. 'No,' he replied, 'it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.'" - G.H. Hardy "As for his place in the world of Mathematics, we quote Bruce C Berndt: ``Paul Erdos has passed on to us Hardy's personal ratings of mathematicians. Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100, Hardy gave himself a score of 25, J.E. Littlewood 30, Hilbert 80 and Ramanujan 100. G.H.Hardy."

3.6 FELLOW OF THE ROYAL SOCIETY (FRS)

Srinivasa Ramanujan was elected a Fellow of the Royal Society (FRS) in 1918, becoming the second Indian to receive this honor. At the age of 31, he was one of the youngest Fellows elected to the Royal Society in its history. His election was for his groundbreakingwork in mathematics, particularly his investigations into elliptic functions and the theory of numbers. At 31 years old, he was one of the youngest individuals to become a Fellow. The Royal Society has hosted events to commemorate the centenary of his election, including a conference in October 2018 and special publications.

3.7 SUMMARY

Dear learners after completing this unit you can say, it has been more than a century, however, the mathematical discoveries of Ramanujan still alive and flourishing. "Ramanujan is important not just as a mathematician but because of what he tells us that the human mind can do”. "Someone with his ability is so rare and so precious that we can't afford to lose them. A genius can arise anywhere in the world. It is our good fortune that he was one of us. It is unfortunate that too little of Ramanujan’s life and work, esoteric though the latter is, seems to be known to most of us". India honour the contributions of Srinivasa Ramanujan by celebrating **National Mathematics Day** on 22nd December every year.

Srinivasa Ramanujan viewed mathematics as a spiritual endeavor, famously stating, "An equation has no meaning for me unless it expresses a thought of God". He believed that his mathematical insights came from divine inspiration, particularly from the goddess Namagiri. For Ramanujan, a beautiful and elegant equation revealed the divine order and was a way to understand God's profound thoughts.

"Infinite series were Ramanujan's first love."

3.8 GLOSSARY

i. Ramanujan life and family
ii. Ramanujan mathematical contribution
iii. Number Theory and Partition Functions
iv. **Continued Fractions and Modular Forms**
v. 1729 Ramanujan Number
vi. G.H.Hardy.

CHECK YOUR PROGRESS

CHQ3.Which film is based on Ramanujan's life?

i. The Man who knew infinity
ii. A Beautiful Mind
iii. Gifted
iv. X+Y

CHQ4. Who invited Ramanujan to England?

i. John Littlewood
ii. G.H. Hardy
iii. Isaac Newton
iv. David Hilbert

3.9 REFRENCES

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3.10 SUGGESTED READINGS

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3.11 TERMINAL QUESTIONS

TQ1. What is the contribution of Ramanujan in mathematician?

……………………………………………………………………………

TQ2. What is the famous quote of Ramanujan?

…………………………………………………………………….

TQ3. Who said Ramanujan is my discovery?

………………………………………………………….

TQ4. Why is 1729 called the Ramanujan number?

…………………………………………………………………

3.12 ANSWERS

CYP1: 22 December 1887.

CYP2: Erode.

CYP3: The Man who knew infinity.

CYP4:G.H. Hardy.

BLOCK- II
VEDIC MATHEMATICS: VEDAS, ŚULVA SŪTRAS, BODHĀYANATHEORUM AND INDIAN ASTRONOMY

UNIT 4- MATHEMATICAL KNOWLEDGE IN THE VEDAS AND ŚULVA SŪTRAS

CONTENTS:

4.1 INTRODUCTION

In previous unit we discussed about Srinivasa Ramanujan. In this unit we explained about Mathematics in ancient India finds its earliest expressions within the sacred texts of the Vedas and their auxiliary treatises, the Śulva Sūtras. It utilizes 16 primary sutra- 13 sub-sutras to tackle various mathematical problems, including arithmetic, geometry, algebra, and calculus. The system arithmetic, geometry, algebra and calculus. The system focuses on mental calculation and aims to simplify complex mathematical processes.

Far from being purely spiritual or liturgical in nature, these ancient Sanskrit scriptures reveal a profound engagement with mathematical thought, woven into rituals, cosmology, and philosophical inquiry. The Vedas contain references to numbers, arithmetic progressions, geometry, fractions, and even abstract concepts such as infinity and zero. Building on this foundation, the Śulva Sūtras—literally "Rules of the Cord"—provide detailed instructions for the geometric construction of ritual altars, introducing methods akin to the Pythagorean theorem, square roots, and precise measurements. Although these texts are not mathematical treatises in the modern sense, they represent a sophisticated and practical application of mathematical knowledge, reflecting a deep understanding of both numerical and spatial relationships. This exploration highlights the significant role of mathematics in Vedic culture and its lasting impact on the development of Indian mathematical traditions.

4.2 OBJECTIVES

After studying this unit learners will able

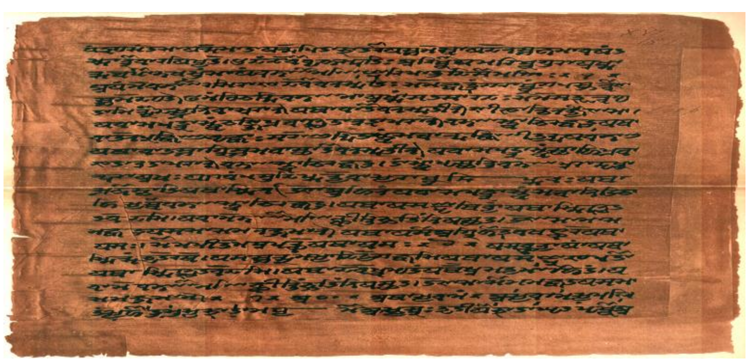
1. To analyze the contributions of the Śulva Sūtras as early mathematical texts, particularly their application of geometric principles and approximation techniques.

2. To highlight the influence of Vedic and Śulva Sūtra mathematics on later Indian mathematical traditions and their relevance to the global history of mathematics.

3. To appreciate the practical and theoretical integration of mathematics in ancient Indian culture and its enduring intellectual legacy.

4.3. MATHEMATICAL REFERENCES IN VEDAS

The **Vedas**—ancient Indian scriptures composed in Sanskrit—contain many early references to **mathematical ideas**, especially in the context of ritual, cosmology, and philosophy. Though they are not mathematical texts in a modern sense, they include concepts and patterns foundational to later developments in **Indian mathematics**.The Vedas, ancient Indian texts, contain mathematical concepts and theories, though not explicitly as a formal mathematical treatise, they do contain references to numbers, fractions, and arithmetic progressions, as well as the concept of infinity. Additionally, the Vedas mentions a system for denoting numbers up to 10^18 and the existence of zero, along with other mathematical ideas like prime numbers and estimations for Pi.



<u>The Vedas are ancient Sanskrit texts of Hinduism (above pic is of Athar Veda) : Pic Credit from Wikipedia (Fig 4.1)</u>

Here are some Specific Mathematical References in the Vedas and related **Vedic literature**:

4.3.1 GEOMETRY ŚULBASŪTRA

"The ŚulbaSūtras, meaning 'Rules of the Cord,' are supplementary texts to the Kalpa Sūtras, which themselves belong to the Vedāṅgas—ancillary disciplines that support the understanding of the Vedas." They are **practical geometry manuals** used for constructing **Vedic fire altars** (yajña-vedis). Notable authors: **Baudhāyana**, **Āpastamba**, **Katyāyana**, **Mānava**.

4.3.2 MATHEMATICAL IDEAS IN ŚULBA SŪTRAS:

Pythagorean theorem (Baudhāyana's version predates Pythagoras):"The diagonal of a rectangle produces both areas which the two sides produce separately."Use of **square roots**, e.g., approximation of $\sqrt{2}$:$\sqrt{2} \approx 1 + 1/3 + 1/(3\times4) - 1/(3\times4\times34) =$ **1.4142156…** (remarkably accurate).

4.3.3 AIRTHMATIC AND ALGEBRAIC IDEAS

Yajur Veda and **Rig Veda** contain references to **large numbers,powers of 10,** and **combinatorics**.Numbers like **10^12 (parardha)** show an awareness of very large values.**Place value system**and**zero** are not explicitly Vedic but evolved later in Indian mathematics (by the time of **Āryabhaṭa** and **Brahmagupta**).

4.3.4 COMBINATORICS AND CHANDAS (PROSODY)

Chandaḥśāstra (science of poetic meters), especially in the **Ṛgveda**, deals with **metrical patterns,** which involve permutations and combinations.

Example: Pingala's ChandasŚāstra (later Vedic) introduces:**Binary representations** (laghu/guru = short/long syllables → like 0 and 1).**Pascal's Triangle** for counting meters — called **Meru-prastaara.Fibonacci numbers**and**combinatorics.**

4.3.5 COSMOLOGICAL NUMBERS

The **Rig Veda** and **Purāṇas** describe cycles of creation with **largetime scales: Yugas, kalpas**, etc., measured in **millions of years**.Cosmological numeracy likely inspired mathematical developments in calendar science and astronomy.

4.3.6 BIG NUMBER

."The Yajurveda presents a sequence of numbers beginning with four and increasing by four each time, forming an arithmetic progression."It also mentions counting numbers up to 10^18, with the highest number being named parardha, according to IJSRD.

4.4 COMPREHENSIVE EXPLORATION OF 16 SUTRAS

The **16 Sutras** and **13 Upa-Sutras** form the core of **Vedic Mathematics**, a system popularized by **Swami Bharati Krishna Tirthaji** in the early 20th century. He claimed these sutras were derived from the **Atharva Veda**, though such direct references are not found in extant Vedic texts. Regardless, these formulas are powerful **mental math tools** that simplify arithmetic, algebra, geometry, and calculus. Below is a **comprehensive exploration** of all **16 Sutras** including their meanings, applications, and examples:

No.	Sutra (Sanskrit)	Meaning	Application Area
1	**EkādhikenaPūrvena**	"Increment by one"	Squaring numbers ending in 5
2	**NikhilamNavataścaramamDaśatah**	"All from 9 and the last from 10"	Fast subtraction and multiplication close to base 10, 100, etc.
3	**Ūrdhva-Tiryagbhyām**	"Vertically and crosswise"	General multiplication, algebra
4	**ParāvartyaYojayet**	"Transpose and adjust"	Solving equations, division
5	**ŚūnyamSāmyasamuccaye**	"When the sum is the same, the sum is zero"	Solving algebraic equations
6	**(Ānurūpyena)**	"Proportionally"	Simplifying ratios, solving proportions
7	**Sankalana-Vyavakalanābhyām**	"By addition and by subtraction"	Solving systems of equations
8	**Pūranāpūranābhyām**	"By completion and non-completion"	Completing the base, multiplication
9	**Chalana-Kalanābhyām**	"By motion and calculation"	Calculus (limited use)

10	**Yaavadūnam**	"Whatever the deficiency"	Multiplication close to a base
11	**Vyashti-Samashti**	"Part and whole"	Partitioning numbers
12	**ŚeṣānyankenaCharamena**	"The remainders by the last digit"	calculations involving remainders.
13	**Sopāntyadvayamantyam**	"The ultimate and twice the penultimate"	Algebra, factorization
14	**EkanyūnenaPūrvena**	"Previous value minus one"	calculations involving numbers and their relationships.
15	**Gunitasamuccayah**	"The product of the sum of the coefficients"	Factoring expressions
16	**Gunakasamuccayah**	"The factors of the sum of the coefficients"	calculations involving factors and sums.

Table 3.3(i) The 16 Vedic Sutra (main Aphorisms)

4.5 COMPREHENSIVE EXPLORATION OF 13 UPA(SUB)- SUTRAS

No.	Sutra (Sanskrit)	Meaning	Application Area
1.	**AnurūpyeŚūnyamanyat**	"If one is in ratio, the other is zero"	Solving equations
2.	**Sankalana-vyavakalanābhyām**	"By addition and by subtraction"	Algebra (repeats Sutra 7)
3.	**Pūrṇāpūrṇābhyām**	"By completeness or incompleteness"	Multiplication (repeats Sutra 8)
4.	**Chalana-kalanābhyām**	"By motion and calculation"	Calculus (repeats Sutra 9)
5.	**YāvadūnamTāvadūnīKṛtvā**	"Whatever the deficiency, subtract that same amount"	Base multiplication
6.	**YāvadūnamTāvadūnīkṛtyaVargaṅchaYojayet**	"Adjust the deficiency and add the square"	Squaring numbers

7.	**Antyayordashake'pi**	"When the last digits add to 10"	Multiplication shortcut
8.	**Antyayoreva**	"Only the last terms"	Algebra, multiplication
9.	**Samucchayagunitah**	"Common factors in sum"	Factoring expressions
10.	**Lopanasthāpanābhyām**	"By elimination and retention"	Solving equations
11.	**Ādyamādyenantyamantyena**	"First by first and last by last"	Binomial multiplication
12.	**KevalaihSaptakamGūṇyah**	**translates to "for 7, the multiplicand is 7."**	**It's used for multiplication tables.**
13.	**Dhvajankaḥ**	"Flag number"	Advanced multiplication, recurring decimals

Table 4.4(ii) The 13 Upa(Sub)- Sutras

Examples and Applications

Sutra1: Ekādhikena Pūrvena

Example: 252=?25^2 = ?252=?

(i) Previous digit = 2
(ii) Add 1 → 3
(iii) Multiply 2 × 3 = 6
(iv) Append 25 → **625**

Sutra2: NikhilamNavataścaramamDaśatah

Example: 98×9798 × 9798×97

(i) Base = 100
(ii) Deficiencies: 2 and 3
(iii) Cross-subtract: 98 - 3 = 95
(iv) Multiply deficiencies: 2 × 3 = 06
(v) Answer: **9506**

Sutra 3:Ūrdhva-Tiryagbhyām (Vertical and Crosswise)

Example:

23×14= (2×1) | (2×4+3×1) |(3×4)=2|11|1223 × 14 = (2×1) | (2×4 + 3×1) | (3×4) = 2 | 11 | 1223×14=(2×1)|(2×4+3×1)|(3×4)=2|11|12

→ **322**

Upa-Sutra 7: Antyayordashake'pi

Example: 43×6743 × 6743×67 (last digits 3 + 7 = 10, tens digits same and add up to 10)

(i) Multiply first digits: 4 × 6 = 24
(ii) Multiply last digits: 3 × 7 = 21
(iii) Final answer: **2881**

4.6 EXTANT ŚULBASŪTRA TEXTS & THEIR COMMENTARIES

The **Śulbasūtras** (or **Śulva Sūtras**) are ancient Indian texts that form part of the **Kalpa Sūtras**, themselves one of the six **Vedāṅgas** (limbs of the Veda). They are **geometrical manuals** dealing with altar construction and include some of the earliest known Indian mathematics. While they are **not philosophical texts**, they reflect a highly practical and rigorous understanding of geometry, including concepts such as the **Pythagorean theorem,** square roots, and area transformations. The extants are mainly those of Baudhāyana, Mānava, Āpastamba, and Kātyāyana along with a few others such as Maitrayana, Varaha and Vadhula. Commantaries are also available on these texts, providing interpretations and explanations.

4.6.1 EXTANT ŚULBASŪTRA TEXTS

There are **four principals extant Śulbasūtras,** named after their authors—Vedic scholars associated with specific **śākhās(recensions)** of the Vedas:

(i) **BaudhāyanaŚulbasūtra:** Baudhāyana Śulbasūtra is Taittirīya-śākhā of the Kṛṣṇa YajurvedaEstimated ~800 BCE or earlier.Its main features is the first record form the Pythagoras Theorum .Its geometrical construction is converting square to the rectangle and circle to the square. Some medieval Indian commentators have glossed the technical terms, but most interpretation is modern.

(ii) **Āpastamba Śulbasūtra:** Āpastamba Śulbasūtrais Taittirīya-śākhā of the Kṛṣṇa YajurvedaEstimated ~600 BCE. Its main features is the more systematic than Baudhtheāyana's, Introduces recursive geometrical method,area-preserving trandformations with higher accuracy first record form the PythagorasTheorum. Its geometrical construction is converting square to the rectangle and circle to the square. Some medieval Indian commentators have glossed the technical terms, but most interpretation is modern and detailed procedures for laying out isosceles trapezia.

Rectangle, squares. Some classical commentaries survive in fragments. Modern scholars such as data and sen have translated and annotated and text.

(iii) **Kātyāyana Śulbasūtra:** Kātyāyana Śulbasūtra is Vājasaneyī-śākhā of the Śuklayajurvedaestimated ~3rd centuryBCE or later. Its main features is that it is More concise and technical than earlier Śulbas. It is focus on refinement and advanced altar designs. It describes more complicated transformations between geometric figures. Historically it has been little studies but in modern times it has been analyzed by Indian and Western historians of mathematics.

(iv) **MānavaŚulbasūtra: Mānava Śulbasūtra is** Mānava-śākhāof the Yajurvedaestimated ~300BCE. Its main features are that it is known for clear and structured altar geometry rules. It is considered to be a later compilation, which consolidates and refines earlier knowledge. Rare; most interpretations are modern and comparative in nature.

(v) **Maitrāyaṇa Śulbasūtra**: A less-studied text, mentioned in some sources as being similar to the MānavaŚulbasūtra.

(vi) **Varāha Śulbasūtra:** This text is mentioned as being in manuscript form.

(vii) **Vādhula Śulbasūtra:** Also mentioned as being in manuscript form.

Commentaries on these Shulba Sutras were written to explain and elaborate the geometrical principle of this text. These commentaries provide insight into the interpretation and application of mathematical knowledge with in the texts.

4.6.2 MODERN COMMENTARIES AND TRANSLATION

Scholar/Work	Contribution
Bibhutibhusan Datta & Avadhesh Narayan Singh(1935)	"A History of Hindu Mathematics" – detailed mathematical analysis
T. A. Sarasvati Amma(1979)	"Geometry in Ancient and Medieval India" – extensive on geometric methods
S. G. Dani	Scholarly papers on Pythagorean theorem in India
Kim Plofker(2009)	"Mathematics in India" – comprehensive treatment of ancient Indian math
S.N. Sen & K.S. Shukla	"BaudhāyanaŚulbasūtra" – critical edition with English translation

4.7 MEANING OF ŚULBASŪTRA

The word **Śulbasūtra शुल्बशूत्र** is derived from two Sanskrit terms:

Śulba(शुल्ब): meaning "cord" or "string", which refers to measurement using a cord. **Sūtra(शूत्र):** meaning "aphorism" or "thread", typically a concise rule or guideline in a larger system of knowledge.So, Śulbasūtra literally means "rules or principles (sūtras) related to the use of the measuring cord (śulba)".**Śulbasūtras** are ancient Indian texts that are essentially geometric guidelines given in the form of sutras (aphoristic statements). The **Śulbasūtras** are a part of the **Kalpa Sūtras**, which are among the **Vedāṅgas** (ancillary disciplines associated with the Vedas). The Śulbasūtras primarily deal with the **geometrical and mathematical rules** used for constructing **altars and sacrificial grounds** (vedis) for Vedic rituals.

Thus, the Shulbasutras can be translated as:“Rules (Sutras) concerning the use of cords (shulbas) for geometrically and constructional purposes.” These include some of the oldest known uses of geometry and concepts such as the Pythagorean theorum.

Notable Śulbasūtras include the following:

(i) Baudhāyana
(ii) Āpastamba
(iii) Kātyāyana
(iv) Mānava

These texts contain astonishingly knowledge, include an approximation of π and the Pythagorean theorem long before Pythagoras.

4.8 QUALITY OF ŚULBAKĀRA (शुल्बकार)

A Shulbakara is a person who practices or writes works related to the Shulba Sutras – essentially a ritual geometer of Vedic alter maker. The following are the qualities of a Shulbakara:

(i) Mastery of Geometry and Measurement: Deep understanding of spatial measurements and geometric constructions. Skilled in using ropes (śulba) and pegs to draw precise shapes like squares, rectangles, trapeziums, and circles.
(ii) Accuracy and Precision: Capable of constructing ritual altars with exact dimensions as prescribed in the Vedic texts. Able to perform transformations (e.g., converting a square to a circle or vice versa) with minimal error.
(iii) Knowledge of Vedic Rituals: Well-versed in the ritual significance of the various altar shapes and arrangements. Understands the spiritual and cosmological symbolism behind altar construction.

(iv) Mathematical Insight: Familiar with the use of irrational numbers, area calculations, and ratios. Applies methods that are precursors to integral and algebraic concepts.
(v) Discipline and Devotion: Performs his duties with great care, as improper construction could be considered ritually inauspicious. Often works under the guidance of Vedic priests, aligning with their spiritual and ritual requirements.
(vi) Oral and Practical Transmission: Proficient in memorizing and reciting sūtras accurately.Experienced in the practical application of sūtra instructions in the physical world.

CHECK YOUR PROGRESS

1. Vedic Mathematics is based on 16 sutras and 13 sub-sutras.
2. Vedic Mathematics is useful only for puzzles and has no practical calculation value.
3. Vedic Mathematics simplifies mental calculations.
4. Vedic Mathematics was reintroduced in the 20th century by Jagadguru Bharati Krishna Tirthaji.
5. Bodhāyana's theorem was used only for constructing squares.
6. Ancient Indian astronomy is known as *Jyotiṣa*.

MULTIPLE CHOICE QUESTIONS

1. What is the approximate value of π (pi) mentioned in the ŚulvaSūtras?
A) 3.14
B) 3.16
C) 3.20
D) 3.25

2. Which ancient Indian text contains mathematical concepts for constructing altars?
A) Vedas
B)ŚulvaSūtras
C) Both A and B
D) Neither A nor B

3. What type of mathematical equations are solved in the Vedas?
A) Linear equations
B) Quadratic equations
C) Both A and B
D) Neither A nor B

4. What is the significance of the ŚulvaSūtras in mathematics?
A) Development of calculus
B) Calculation of π (pi)
C) Geometric constructions
D) All of the above

5. Which mathematical concept is demonstrated in the ŚulvaSūtras?
A) Pythagorean theorem
B) Algebraic manipulations
C) Geometric calculations
D) All of the above

4.9 SUMMARY

Vedic texts and Shulba Sutras contain detailed descriptions of the mathematical knowledge of ancient India. The Vedas (especially the Rig Veda, Yajur Veda and Atharva Veda) contain references to numbers, calculation, geometry and measurement. These were mainly used in Yagya, astronomy and construction of altars. Use of large numbers, fractions, ratios and basic mathematical operations is also found. The Shulba Sutras (composed between c. 800 BCE and 200 BCE) are essentially a collection of geometrical rules for the construction of sacrificial altars. These include advanced mathematical knowledge like Pythagoras theorem, construction of shapes like square, rectangle, circle etc. and approximation of $\sqrt{2}$. Thus, the Vedas and Shulba Sutras show that mathematics in ancient India was not only theoretical but was also deeply connected to practical life, Vastu and Astronomy. This later became the foundation for the development of algebra, arithmetic and geometry. Sulba Sutras contain advanced geometry, such as the Pythagorean theorem, square-rectangle construction and approximation of $\sqrt{2}$. This mathematics was related to practical life, architecture and astronomy.

4.10 GLOSSARY

i. **Pythagorean theorem:** First stated geometrically: "The diagonal of a rectangle produces both areas"
ii. **Geometric transformations:** Converting shapes: square → rectangle, triangle → square, etc.
iii. **Approximation techniques:**Approximate values of $\sqrt{2}$, π, and others
iv. **Proportional altar design:**Prescribed using precise ratios (e.g., 1:2, 3:4)
v. **Use of cords (śulba):** Geometry done via rope-stretching, hence the name *Śulbasūtra*
vi. **Mathematical Concepts:**The Śulbasūtras contain mathematical concepts related to the Pythagorean theorem, area calculations, and circle squaring.
vii. **Ancient Indian Mathematics:**These texts are considered to be the only sources of knowledge of Indian mathematics from the Vedic period.

4.11 REFERENCES

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5. Dharampal,*Some Aspects of Earlier Indian Society and Polity and Their Relevance Today*, New Quest Publications, Pune, 1987.

4.12 SUGGESTED READING

1. **"Vedic Mathematics" by Bharati Krishna Tirthaji:** The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras.Covers mental math techniques for arithmetic, algebra, and geometry
2. **"Vedic Mathematics Made Easy" by Dhaval Bathia :** A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.
3. **"The Power of Vedic Maths" by Atul Gupta:** Focuses on speed and accuracy.Ideal for practical application in exams like CAT, GRE, and banking tests.
4. **"Mathematics in India" by Kim Plofker:** A scholarly account of the historical development of mathematics in India.Provides context on how Vedic methods evolved over time.
5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

4.13 TERMINAL QUESTIONS

1. To what extent are the 16 Sutras and 13 Upa-Sutras of Vedic Mathematics authentically rooted in the ancient Vedas, and how should their mathematical significance be interpreted in modern context?

2.How do the mathematical concepts in the extant Śulbasūtras—particularly in their treatment of geometry, irrational numbers, and construction principles—reflect an indigenous epistemology distinct from later formalized mathematical systems, and what insights can their commentaries offer into the oral-ritual transmission of mathematical knowledge in Vedic India?

3.Analyze the contributions of the Vedas and ŚulvaSūtras to the development of mathematics in ancient India, highlighting their significance in geometry, arithmetic, and algebra.

4.What qualities and skills would a Śulbakāra (Vedic architect/mathematician) need to possess in order to accurately construct intricate Vedic altars and sacred spaces, adhering to precise geometric and mathematical principles?

4.14 ANSWERS

CYQ1. True CYQ2.False CYQ3. True CYQ4.True CYQ5.False CYQ6.True

MCQ 1. B MCQ 2. C MCQ3. C MCQ4. C MCQ5. D

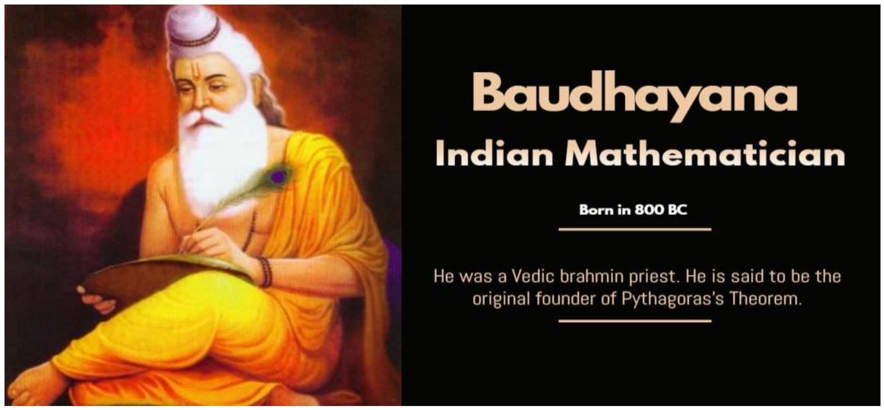
UNIT 5 - BRIEF INTRODUCTION ABOUT BODHAYANA THEORUM

CONTENTS:

5.1INTRODUCTION

Bodhāyana's was an ancient Indian mathematician and Vedic scholar who lived around the 8th-6th century BCE. He is best known for composing the Bodhāyana's Śulbasūtra., a mathematical text that describes the rules for constructing altars and geometric figures. This text provides mathematical formulas and techniques for constructing various geometric shapes, including squares, rectangles, and triangles. Bodhāyana's work includes a statement of the Pythagorean theorem, which describes the relationship between the sides of a right-angled triangle. Bodhāyana'sShulba Sutra provides methods for constructing various geometric shapes, including squares, rectangles, and triangles, using mathematical formulas and techniques. He is most famous for the *Bodhāyana's Śulbasūtra*, one of the oldest texts in the Śulbasūtra Bodhāyana's theorem refers to an ancient statement of the Pythagorean theorem found in Indian mathematical texts, attributed to the sage Baudhayana (also spelled Bodhayana). He composed the Bodhāyana'sShulba Sutra around the 8th-6th century BCE, a manual describing the rules for constructing altars and geometric figures.

In previous block we have studied the Introduction and History of Vedic Mathematics present unit emphasizes that **Bodhāyana's theorem** is a cornerstone of **Vedic geometry**, showing both the mathematical depth and spiritual context of ancient Indian science. It reveals how geometry was an essential part of religious and ritual practice, and how Indian mathematicians were deeply engaged in spatial reasoning centuries before similar ideas appeared in Greece.



Pics credit: Vedic math school Fig 5.1

5.2 OBJECTIVES

After studying this unit learner will be able

1. To explain who Bodhāyana'swas and describe his contributions to ancient Indian mathematics, especially through the Śulbasūtras.
2. To identify right-angled triangles and apply the theorem to calculate the length of an unknown side.
3. To discuss the historical significance of Bodhāyana's theorem in the development of geometry worldwide.
4. To recognize examples of right-angled triangles and verify Bodhāyana's theorem with numerical calculations.

5.3 FINDING THE CARDINAL DIRECTIONS

The theorem itself has nothing to do with directions like north, south, east, or west.It only relates lengths of the sides of a right-angled triangle:

$$(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$$

Using geometry for navigation or orientation, here's how ancient methods sometimes connected them:

(1)**Gnomons & shadows**: In Vedic and ancient Indian traditions, right-angled triangles were used in sundials or gnomons to determine cardinal directions. By measuring the shadow of a vertical stick (gnomon) at solar noon (when the sun is at its highest point), the shortest shadow points due north (in the Northern Hemisphere). The perpendicular to this line gives east-west.

(2) Constructing cardinal directions geometrically: A practical way to lay out north-south-east-west on flat ground:

(i) Place a stick vertically in the ground.
(ii) Draw a circle around the stick's base.
(iii) Mark the point where the tip of the morning shadow touches the circle — that's roughly **west**.
(iv) Mark the point where the afternoon shadow touches the circle — that's roughly **east**.
(v) The line connecting east and west point's gives the east-west axis; a perpendicular line through the stick's base gives the north-south axis.

This process inherently uses right-angled triangles formed by the stick, its shadow, and the distance along the ground **Bodhāyana's theorem** can help calculate precise lengths if you need accurate measurement.

5.4 METHODS FOR PERPENDICULAR BISECTOR

The method for constructing a perpendicular bisectorof a line segment (known since ancient times and relevant in Bodhāyana'sgeometric constructions) is simple and can be done with just a compass and straightedge. Here's a clear step-by-step guide:

Steps to construct a perpendicular bisector of a line segment AB:

(i) Draw the line segment AB. First, we Mark points A and B.
(ii) Set your compass wider than half the length of AB.
(The compass width must be more than half, but doesn't need to be exact.)
(iii)From point A: Draw an arc above and below the line segment.
(iv)From point B: Without changing the compass width, draw arcs that intersect the arcs from point A above and below the line.
(v) Mark the two intersection points of the arcs above and below the line as points C and D.
(vi)Draw a straight line through points C and D.
This line is the **perpendicular bisector** of AB. It meets AB exactly at its midpoint and is perpendicular (90°) to AB.

Properties of the perpendicular bisector:
It divides AB into two equal parts.
Any point on the bisector is equidistant from A and B.

The intersection point is the midpoint of AB, which can be labeled M.

5.5 BODHĀYANA'S METHOD OF CONSTRUCTING A SQUARE

Bodhāyana's method for constructing a square geometrically is one of the earliest known contributions to geometry, found in the **Śulbasūtras**, ancient Indian texts used for altar constructions. It predates even Euclid and forms a foundational part of Vedic mathematics. Here's how Bodhāyana's method works:

Bodhāyana's method of constructing a square is a significant concept from **Vedic mathematics** and ancient Indian geometry. Bodhāyana was an ancient Indian mathematician and the author of the **Śulbasūtras** (circa 800 BCE), which are among the earliest texts to discuss geometric constructions, including techniques related to the Pythagorean theorem.

BODHĀYANA'S CONSTRUCTION OF A SQUARE EQUAL IN AREA TO A GIVEN RECTANGLE

This is a more advanced idea found in the **ŚulbaSūtras**, often referred to as the **squaring of a rectangle** (or even a circle in some interpretations).The construction of a square from a given area or side length — and transforming one square into another — is a core geometric idea in the **ŚulbaSūtras**. Below is a classical geometric method Bodhāyana used:

1. CONSTRUCTING A SQUARE ON A GIVEN LINE SEGMENT

Let's say you are given a line segment AB and need to construct a square on it.

Steps (Geometric Construction):

1. **Draw segment AB** of desired length.
2. Using a compass:
   (i) Construct a perpendicular at point A.
   (ii) On the perpendicular, mark point D such that AD = AB.
3. Using the compass, draw an arc from point B with radius equal to AB.
4. With the same radius, draw an arc from point D intersecting the previous arc at point C.
5. Connect points B to C, C to D, and D to A to form square ABCD.

This method relies on **geometric construction using only a straight edge and compass**, and it reflects the practical geometry known to Vedic scholars.

2. CONSTRUCTING A SQUARE EQUAL IN AREA TO A GIVEN RECTANGLE

This is a more advanced idea found in the **ŚulbaSūtras**, often referred to as the **squaring of a rectangle** (or even a circle in some interpretations).

Given: A rectangle with sides **a** and **b**.

Goal: Construct a square with the same area (i.e., side = √ (a × b)).

Method:

1. Draw a rectangle with sides **a** and **b**.
2. Use geometric mean construction:
   (i) Extend side **a** and mark point so the full length is **a + b**.
   (ii) Find the midpoint of the segment (a + b).
   (iii) Draw a semicircle with this midpoint as center.
   (iv) From the junction point of a and b, erect a perpendicular to the semicircle.
   (v) The length of the perpendicular is √(ab) — the side of the square.

This uses the **geometric mean theorem** and is a very advanced concept for its time.

Relation to Pythagorean Theorem

Bodhāyana also stated a form of the Pythagorean Theorem:

(Śulba Sūtra: dīrghasyākṣaṇayā rajjuḥ pārśvamānī tiryaṅmānī ca yat pṛthagbhūte kurutastad ubhayāṅ karoti)

दीर्घस्याक्षणया रज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयाङ्करोति

"आयत की कर्ण रेखा (आयताकार आकृति), लंबाई और चौड़ाई द्वारा अलगअलग बना-ए गए वर्गों के क्षेत्रफलों के योग के बराबर क्षेत्रफल उत्पन्न करती है।

"The diagonal of a rectangle produces both areas which the two sides produce separately"

This is essentially: If the side are a and b, then the diagonal c satisfies $a^2 + b^2 = c^2$

5.6 BODHĀYANA'S THEORUM

Bodhāyana's theorem is an ancient Indian statement of what we now call the Pythagorean theorem. It explains a special relationship between the three sides of a right-angled triangle.The theorem states that

"In **a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides."** This is the same relationship described by the Pythagorean theorem in

Greek mathematics. The original Sanskrit sutra from the **Śulbasūtra** says: "दीर्घचतुरस्रस्याक्षणया रज्जुः पार्श्वमानी तिर्यग्मानी च यत् पृथग्भूते कुरुतः तदुभयं करोति।"

It means:“The rope stretched along the diagonal of a rectangle makes an area which the horizontal and vertical sides make separately.”

In his **Shulba Sutra,** he stated:

“The diagonal of a rectangle produces both areas, which the two sides make separately.” In modern language this is the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

Where a and b are the two legs of the right-angle triangle. And c is hypotenuse. Bodhāyana introduced the theorem as part of practical geometry (Śulba-vidhi) needed for constructing yajña altars with precise shapes and sizes. Bodhāyana also gave numerical examples, such as calculating the diagonal of squares with sides 3 and 4 to get 5—the classic 3-4-5 triangle. This shows that knowledge of the relationships between the sides of right-angled triangles existed in India long before Pythagoras, reflecting the independent and advanced development of geometry in Vedic mathematics.

Example given by Bodhāyana:

For a triangle with side 3 and 4 unit

$3^2 + 4^2$ = 9 +16 = 25

So the diagonal (hypotenuse) is $\sqrt{25}$. *Giving the* famous 3-4-5 right angle.Bodhāyana’s work predates Pythagoras by several centuries and shows how ancient Indian mathematicians understood geometry deeply, especially in the context of constructing Vedic fire altars with precise dimensions.

5.7 APPLICATION OF BODHĀYANA’S THEORUM

Baudhayana’s Theorem, an early expression of the Pythagorean Theorem found in the **ŚulbaSūtras** (ancient Indian texts on geometry), plays a significant role in Vedic Mathematics and Vedic practices**,** especially in geometry, altar construction, and symbolic measurements. This theorem explains the relationship between the sides of a right-angled triangle. It states that the square of the hypotenuse—the side opposite the right angle—is equal to the sum of the squares of the other two sides. It has several important practical and theoretical applications across various fields: such as engineering, architecture, astronomy, and surveying.

Shulbasutras: Baudhāyana theorem is used in the Shulbasutras, which are an important part of Vedic Mathematics.

Construction of altars: Thistheorem is used in the construction of altars, which are important in Vedic Mathematics. Baudhayana's theorem was originally used to construct square and rectangularfire altars with precise measurements. It is used to transform one geometric shape into another (e.g., square to rectangle or trapezium) while maintaining the same area.

Example: Squaring the circle or forming complex altar shapes using only ropes and pegs based on the theorem.

Geometrical Calculation: The Baudhāyana theorem is used in geometrical calculation, such as calculating the area and perimeter of triangles and rectangles. It was used to divide and combine squares and rectangles in sacred geometric patterns. Essential for area-preserving transformations, a key aspect of Vedic altar design rituals. It also Helped in visual and rope-based methods (without modern tools) to solve geometrical problems.

Foundation of later Vedic geometry: Baudhayana's work laid the groundwork for later developments in Vedic Geometry, influencing how learners learned mathematical ideas in ancient Gurukulas. "Formed the foundation for teaching reasoning and demonstration, which was a crucial element of ancient Indian mathematical philosophy."

Symbolic and Ritual Use: The accuracy of altar shapes and dimensions has deep ritual and cosmological significance in Vedic culture. It is also Used in astrological and astronomical altar placements to align with cosmic forces.

Emergence of foundational algebraic ideas. The theorem represents algebraic relationships in geometric form, forming the base for higher concepts in Vedic mathematics like quadratic equations and rational approximations.

Land survey: This theorem is used in land survey, where it helps in calculating the area and boundaries of land.

Science and engineering: This theorem is used to calculate the slop of road, roof and other inclined surfaces insuring proper drainage and stability.

CONSTRUCTING A SQUARE THAT IS THE DIFFERENCE OF TWO SQUARES.

In Vedic Mathematics, the "difference of squares" concept is rooted in both algebraic and geometric principles. Ancient Indian mathematicians like Bodhāyana explored geometric interpretations of this concept, which are reflected in the ŚulbaSūtras. These texts provided guidelines for constructing altars and other geometric shapes, demonstrating the practical application of mathematical concepts. The algebraic identity $a^2 - b^2$ = (a+b) (a-b) tell us that the difference of two square.

Geometrical construction of $a^2 - b^2$ in Vedic

Suppose we have given two squares. One is the large square which side is a and another is the smaller square which side is b. such that a is greater than b. Our aim is to construct a square whose area is $a^2 - b^2$. Now we are giving steps for Geometrical construction

STEPS FOR GEOMETRICAL CONSTRUCTION

Step I: Draw the large square Here we are drawing a square whose side is a. This square represent area $2a^2$.

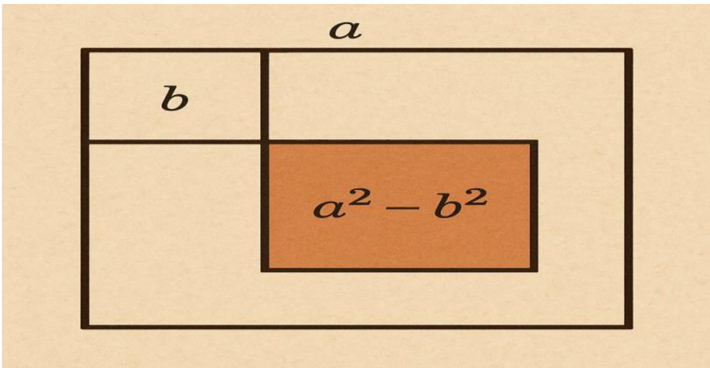
Step II: Draw the smaller square inside it: Inside this square at one corner, draw a square of side b. this square represent area b^2.

Step III: Remove the smaller square: Now imagine removing (or cutting off) the smaller square from the corner. The remaining area is not a square yet — it's an L-shaped figure.

Step IV: **Rearranging the remaining area:** Rearrange this L-shaped area into a new square. In Vedic methods (and the ŚulbaSūtras), this is done by cutting and rejoining pieces.

VEDIC INTERPRETASTION

The difference in two square areas can be transformed into another perfect square by rearrangement. This method shows **area conservation.** This method creates a beautiful connection with the algebraic identity $(a + b)(a - b) = a^2 - b^2$, which becomes even more evident through geometric interpretation. The rectangle formed by sides a+b and a-b also has the same area as the difference of squares and from that, a square can be constructed using geometrical methods.



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CHECK YOUR PROGRES

Multiple choice question

1. **Who was Bodhāyana?**
   a) Greek philosopher
   b) Chinese astronomer
   c) Ancient Indian mathematician
   d) Roman architect
2. **Bodhāyana's theorem is an early statement of which modern mathematical principle?**
   a) Law of Sines
   b) Pythagorean Theorem
   c) Area of a Circle
   d) Volume of a Cylinder
3. **In which ancient text is Bodhāyana's theorem found?**
   a) Arthashastra
   b) Rigveda
   c) ŚulbaSūtras
   d) Manusmriti
4. **What shape is primarily involved in Bodhāyana's geometric constructions?**
   a) Circle
   b) Triangle
   c) Square
   d) Rectangle
5. **What does Bodhāyana's theorem state about the diagonal of a rectangle?**
   a) It is equal to the sum of the sides
   b) It produces the sum of the squares of the sides
   c) It is longer than any side
   d) It divides the rectangle into two equal parts
6. **The ŚulbaSūtras were primarily written for which purpose?**
   a) Teaching school mathematics
   b) Recording astronomical data
   c) Constructing ritual fire altars
   d) Writing poetry

5.7 SUMMARY

In Vedic Mathematics, Baudhayana's Theorem is more than a mathematical formula—it is a practical tool, a spiritual guide, and a mathematical foundation rooted in ancient Indian wisdom and geometry. The unit introduces **Bodhāyana**, an ancient Indian mathematician and sage, who is credited with one of the earliest known statements of the Pythagorean Theorem. This theorem is found in his work, the ŚulbaSūtras**,** ancient texts dealing with the geometric principles required for Vedic altar constructions. Bodhāyana's theorem states that:"The diagonal of a rectangle produces both areas which the two sides produce separately."This is essentially the same as the modern Pythagorean Theorem**.**

Geometric Methods: Bodhāyana used geometrical constructions rather than algebra to explain his results — a feature typical of early Vedic mathematics.

5.8 GLOSSARY

i. **Pythagorean theorem:** Found in Śulbasūtras
ii. **Theorem:** A mathematical statement that can be proven based on previously established principles.
iii. **Śulbasūtras:** Ancient Indian texts containing rules for constructing altars using geometry.
iv. **Construct square on a line:** Use compass and perpendicular constructions
v. **Construct square equal to rectangle**: Use geometric mean (√ab) method.
vi. **Ritual Altars:** Sacred platforms used in Vedic rituals, often designed using precise geometry.
vii. **Vedic Period:** Historical period in ancient India when the Vedas were composed (around 1500–500 BCE).

5.9REFERENCES

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3. D.M.Bose, S.N.Sen and B.V. Subbarayappa, Eds., *A Concise History of Science in India*, 2nd Ed., Universities Press, Hyderabad, 2010.
4. *Astāngahrdaya*,Vol.I, *Sūtrasthāna* and *Śarīrasthāna*, Translated by K. R. Srikantha Murthy, Vol. I, Krishnadas Academy, Varanasi, 1991.
5. Dharampal,*Some Aspects of Earlier Indian Society and Polity and Their Relevance Today*, New Quest Publications, Pune, 1987.

5.10 SUGGESTED READING

1. **"Vedic Mathematics" by Bharati Krishna Tirthaji:** The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras. Covers mental math techniques for arithmetic, algebra, and geometry
2. **"Vedic Mathematics Made Easy" by Dhaval Bathia :** A simplified, beginner-friendly version of Vedic techniques. Great for learners and competitive exam aspirants.
3. **"The Power of Vedic Maths" by Atul Gupta:** Focuses on speed and accuracy. Ideal for practical application in exams like CAT, GRE, and banking tests.
4. **"Mathematics in India" by Kim Plofker:** A scholarly account of the historical development of mathematics in India. Provides context on how Vedic methods evolved over time.

5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

5.11 TERMINAL QUESTIONS

1. Who was Baudhayana, and what is he famous for in mathematics?
2. State Baudhayana's theorem in your own words.
3. Write the modern mathematical formula for Baudhayana's theorem.
4. Discuss the historical importance of Baudhayana's theorem and how it shows that ancient Indians knew the Pythagorean relationship before Pythagoras.
5. Describe how Baudhayana's theorem can be practically applied in fields such as architecture or navigation.

5.13 ANSWERS

MCQ1. C
MCQ2. B
MCQ3. C
MCQ4. B
MCQ5. B
MCQ6.C

UNIT-6
INDIAN ASTRONOMY AND VEDIC MATHEMATICS

CONTENTS:

6.1 INTRODUCTION

Dear learners, this unit presented the contributions Vedic Mathematics and Indian Astronomy. The unit explores the historical context, foundational principles and contemporary relevance of these ancient disciplines. Vedic Mathematics, rooted in the Vedas, offers a unique approach to mathematical calculations, emphasizing simplicity and mental agility. Vedic Astronomy, on the other hand, reflects the advanced understanding of celestial phenomena by ancient Indian scholars. The unit also explains the interplay between these disciplines and their impact on modern science and education.

Indian mathematicians and astronomers made groundbreaking contributions that laid the foundation for modern science. The “Sulba Sutras” (800–500 BCE) contain some of the earliest known geometric principles, while Aryabhata’s work on trigonometry and calculus predates European discoveries by centuries. The “Surya Siddhanta” provides accurate calculations of planetary motion and eclipses, demonstrating the advanced astronomical knowledge of ancient India.

Among its many facts, Vedic Mathematics and Astronomy stand out as remarkable achievements that continue to inspire scholars and practitioners worldwide. Vedic Mathematics, characterized by its simplicity and efficiency, offers alternative methods for solving complex mathematical problems.

Vedic Astronomy, on the other hand, showcases the advanced understanding of celestial phenomena by ancient Indian scholars, who made significant contributions to the fields of cosmology, calendrics and observational astronomy.

6.2 OBJECTIVES

After studying this unit learner will be able to:

1. Explain the concept of solar system and Vedic Mathematics.
2. Evaluate the diameter of sun, moon and earth with the help of Vedic Mathematics.
3. Describe Nakaṣtras, zodiac signs and eclipse

6.3 AKASHGANHA

In Vedic contexts, "Akash Ganga" refers to the Milky Way galaxy, The term, meaning "the Ganges of the sky," is a poetic name given to our galaxy in ancient India, likening its appearance to a flowing river of light. It forms the abdomen of the “Dolphin”, according to scriptures.

Fig 11.3.1.*Akash Ganga*

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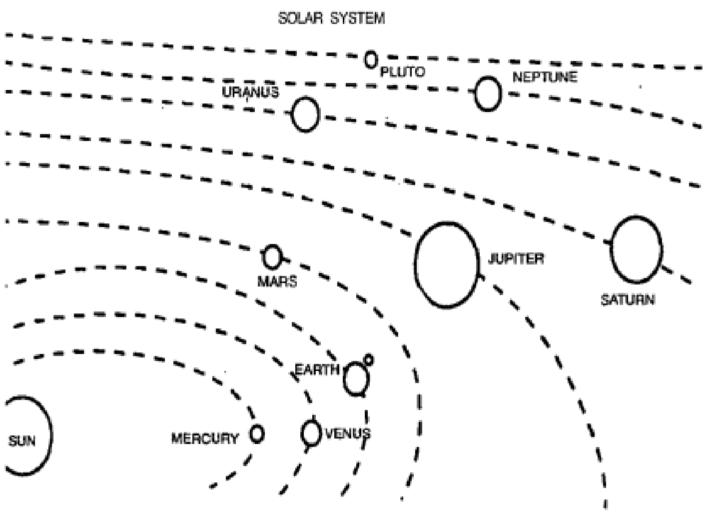
6.4 SOLAR SYSTEM



Solar System
Fig.11.4.1

In Vedic mathematics and astronomy, the solar system is implicitly understood through the concept of the Navagraha, which represents the nine celestial bodies considered influential in Hindu astrology and cosmology.

These include the Sun (Surya), Moon (Chandra), Mars (Mangala), Mercury (Budha), Jupiter (Brihaspati), Venus (Shukra), and Saturn (Shani), along with the ascending and descending lunar nodes (Rahu and Ketu).

While not explicitly detailed as a solar system in the modern sense, the Navagraha framework acknowledges the interconnectedness and influence of these celestial bodies on Earth and human affairs.

- **Navagraha and the Solar System:** The Navagraha system, while not a precise planetary model like the modern solar system, acknowledges the Sun, Moon, and other planets as significant entities influencing life on Earth.
- **Vedic Texts and Astronomy:** Texts like the Rig Veda and Surya Siddhanta contain astronomical observations and calculations related to the movement of celestial bodies, including the Earth's rotation and the apparent motion of the Sun and Moon.
- **Astronomy in Vedic Literature:** The Rig Veda, considered one of the oldest texts, describes the Earth's rotation and the relative positions of planets, indicating an early understanding of the cosmos.

- **Vedanga Jyotisha:** This treatise focuses on astronomy, specifically the solstices and the motion of the Sun and Moon, further demonstrating the depth of ancient Indian astronomical knowledge
- **Surya Siddhanta:** This text provides methods for calculating planetary positions, eclipses, and even the Earth's diameter and circumference, showcasing a sophisticated understanding of celestial mechanics.

In the Vedic literature, there are many references to planetary movements. Rig Veda (the oldest of them) dated back to 1500 BC contains a hymn that describes the movements of Sun, Moon and other planets around Earth. This hymn was discovered in Arabia recently. Along with this there are other references in Vedic literature that describe the motion of other planets around their own axes while moving around Earth at same time they rotate around Sun in fixed positions relative to each other as per Newton’s law on gravitation which states that two objects attract each other by force which is directly proportional to product of their masses and inversely proportional square of distance between them which means gravity decreases as distance increases if same amount of matter present in both bodies then gravitational attraction between them is same irrespective how far away each one might be from earth or sun or any other object .

The Rig Veda is the oldest collection of sacred hymns known to man. It consists of more than a thousand hymns divided into ten mandalas (books).

The Vedic sages have given us precise dates for all planets including our solar system and Earth’s rotation around it. These are given by their observations through telescopes much before Galileo **(15 February 1564 – 8 January 1642),** ever invented one. The Rig Veda mentions that our planet Earth rotates on its axis every 24 hours, which means we see different parts of our planet as we rotate around Sun every day with respect to other planets in our solar system.

Rig Veda 10.22.14

अहस्ता यदपदी वर्धत क्षाः शचीभिर्वेद्यानाम् ।

शुष्णं परि प्रदक्षिणिद्विश्वायवे नि शिश्नथः ॥

When the earth which has neither hands nor feet flourished thought the acts of (devotion paid to) the adorable (deities), then you did smite down Śuṣṇa, circumambulating it on the right, for the sake of Viśvāyu.

“This earth is devoid of hands and legs, yet it moves ahead. All the objects over the earth also move with it. It moves around the sun”

Rig Veda 10.149.1

सविता यन्त्रैः पृथिवीमरम्णादस्कम्भने सविता द्यामदंहत् ।

अश्वमिवाधुक्षद्धुनिमन्तरिक्षमतूर्ते बद्धं सविता समुद्रम् ॥

"Savitā (Refers to god sun) has fixed the earth with fetters; Savitā has made the heaven firm in a plural ce where there was no support; Savitā has milked the cloud of the firmament bound to the indestructible (ether) like a trembling horse."

"The sun has tied Earth and other planets through attraction and moves them around itself as if a trainer moves newly trained horses around itself holding their reins."

Rig Veda 1.35.9

हिरण्यपाणिः सविता विचर्षणिरुभे द्यावापृथिवी अन्तरीयते ।

अपामीवां बाधते वेति सूर्यमभि कृष्णेन रजसा द्यामृणोति ॥

"The gold-handed, all-beholding, Savitā (Refers to god sun) travels between the two regions of heaven and earth, dispels diseases, approaches the sun, and overspreads the sky with gloom, alternating radiance."

"The sun moves in its own orbit but holding earth and other heavenly bodies in a manner that they do not collide with each other through force of attraction".

Rig Veda 1.164.13

पञ्चारे चक्रे परिवर्तमाने तस्मिन्ना तस्थुर्भुवनानि विश्वा ।

तस्य नाक्षस्तप्यते भूरिभारः सनादेव न शीर्यते सनाभिः ॥

"All beings abide in this five-spoked revolving wheel; the heavily-loaded axle is never heated; its eternal compact nave is never worn away."

"Sun moves in its orbit which itself is moving. Earth and other bodies move around sun due to force of attraction, because sun is heavier than them".

Yajur Veda 33.43

आ कृष्णेन रजसा वर्त्तमानो निवेशयन्नमृतं मर्त्यं च।
हिरण्ययेन सविता रथेना
देवो याति भुवनानि पश्यन्॥

"The Sun God Savitar, revolving in his path full of black darkness, moves on his golden chariot watching the worlds. The God Savitar moves, putting men to work".

"The sun moves in its own orbit in space taking along with itself the mortal bodies like earth through force of attraction."

Rig Veda is the oldest book known to man. It describes our Solar System in great detail, and includes information about the Sun, Earth and other planets.

The Rig Veda consists of 1028 hymns divided into 10 mandalas (chapters) consisting of 1008 hymns each – totaling 10,600 verses.

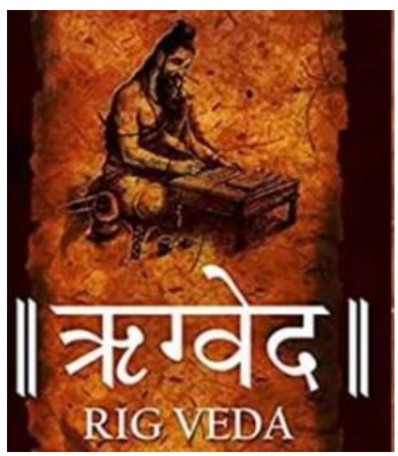


Fig.11.4.2

Ref: https://ebnw.net/empowerment/inspirations/five-reasons-content-indias-oldest-literature-rig-veda-known/

6.4.1 SUN

Surya (Sun): The sun, or surya, is an important symbol of light and illumination in Hinduism. The sun is often associated with knowledge, enlightenment, and the power of consciousness to dispel darkness and ignorance. Many Hindu rituals and practices are performed at sunrise and sunset, when the light of the sun is believed to be most potent.

The Sun, along with the entire solar system, orbits the center of the Milky Way galaxy. This galactic orbit takes the Sun approximately 230 million Earth years to complete. The Sun's speed in this orbit is around 828,000 kilometers per hour. While the planets in our solar system orbit the Sun, the Sun itself is also in motion around the galactic center.

Sunrise to sunset is the route the Sun Planet takes in his giant golden chariot harnessed to seven horses. The chariot of the Sun-god (Surya), worshiped by the words om bhurbhuvasvah, travels at a speed of 3,400,800 yojanas [27,206,400 miles] in a muhurta (a Muhurta equals approximately 48 minutes).

6.4.2 EARTH

The Earth is ever spinning on its axis. In addition to its spinning, the earth is also revolving round the Sun. It is therefore always in a state of motion in the space at a speed of nearly 30 kms per second or 1,800 kms a minute or 9,46,08,000 kms per year.

In early times, the earth was believed to be the centre of universe of our solar system. It was thought that the Sun, the Moon and other planets (stars) actually revolved around the earth, as they appear to do. But now we know that the earth is a globe, that it rotates or spins on its axis and the Sun and stars appear to revolve around it from east to west, because the earth is revolving around its axis from west to east. In living on the earth's surface, we also keep on moving in the space with the same speed as that of the earth. The Sun which is actually stationary would appear to us to be moving in the opposite direction to that of the earth. As the earth is moving from west to east, the Sun and other stars in the space will appear to be moving in the opposite direction i.e. from east to west. This is what we actually observe also.

The axis of the Earth slants at an angle of about $\frac{23^\circ}{4}$ from the perpendicular to the plane of its orbit. If the plane of orbit of earth is treated as horizontal, then perpendicular to this line will be known as vertical and then the axis of earth can be stated to be slanting at an angle of about $23\frac{1}{2}^\circ$ (23 degrees 28 minutes to be precise) to the vertical.

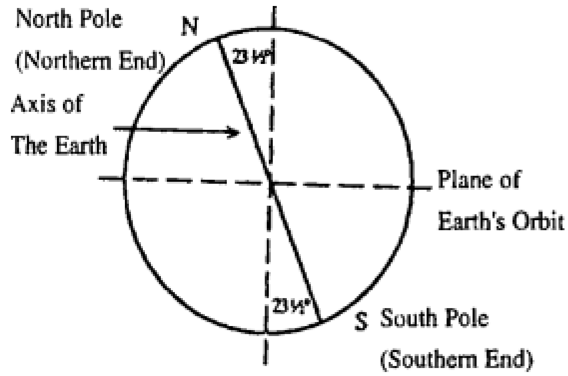


Fig.11.4.2

The axis of the earth is so inclined that the northern end of the axis always points to the Pole Star or commonly known as Dhruv Tara. Where the northern and southern end of the axis of earth meet the surface of the earth, those points are known as North and South Poles of the earth, respectively.

6.4.3 MOON

In Vedic culture, the Moon is primarily a celestial body with astronomical and astrological significance. Vedic mathematics, which involves precise calculations for astronomical phenomena like eclipses and planetary positions, uses the Moon in its calculations for practical purposes like tracking lunar calendars and preparing horoscopes. Numerology assigns the number 2 to the Moon, linking it to individual numbers and birth dates to reveal personality traits and future influences.

Aryabhata, in his work Aryabhatiya, clearly stated that the Earth rotates on its axis, causing the apparent movement of stars and the sun.

6.4.4 DIAMETER OF THE SUN

Suryasiddhant Chapter 1 :59

योजनानि शतान्यष्टौ भूकर्णो द्विगुणानि तु।
तद्वर्गतो दशगुणात् पदं भूपरिधिर्भवेत्।।

The diameter of the sun's disk is six thousand five hundred yojanas ; of the moon's, four hundred and eighty.

Note: One yojana is not a fixed value, but it generally ranges from approximately 8 kilometers to 16 kilometers (5 to 10 miles), depending on the specific historical text, astronomical tradition, or regional context it's being measured by. While ancient texts like the Surya Siddhanta and Aryabhatiya define it around 8 km, later texts and traditions use larger values, sometimes up to 13 km or more.

6.4.5 DIAMETER OF THE EARTH AND MOON

Suryasiddhant Chapter 4 :1

सार्धानि षट्सहस्राणि योजनानि विवस्वतः । विष्क-
म्भो मण्डलस्येन्दोः सहाशीत्या चतुःशतम् ॥१॥ स्फुट-
स्वभुक्त्या गुणितौ मध्यभुक्त्योद्धृतौ स्फुटौ ।

The diameter of the earth being, as stated above 1000 yojanas, that of the moon, 480 yojanas, is 0.3 of it: while the true value of the moon's diameter in terms of the earth's is .2716, or only about a tenth less.

An estimate so nearly corfect supposes, of course, an equally correct determination of the moon's horizontal parallax, distance from the earth, and mean apparent diameter. The Hindu valuation of the parallax may be deduced from the value given just below , of a minute on the moon's orbit, as 15 yojanas.

In Vedic traditions, the number 108 is central to astronomical descriptions, relating the distances and diameters of the Earth, Moon, and Sun. According to these concepts, the distance from the Earth to the Sun is approximately 108 times the Sun's diameter, and the distance from the Earth to the Moon is also about 108 times the Moon's diameter. This cosmological connection, often seen as a significant coincidence, suggests a deep symbolic understanding of the cosmos within Vedic thought, where this number appears frequently in different contexts, like the beads of a mala, representing the journey from the body to the inner self.

6.5 EVOLUTION OF VEDIC THOUGHT

How did the use of altars for a symbolic representation of knowledge begin?

This development is described in the $Pur\bar{a}ns$ as where it is claimed that the three altars were first devised by the king $\bar{u}\,ravas$. The genealogical lists of the $Pur\bar{a}ns$ and the epics provide a framework in which the composition of the different hymns can be seen.

The ideas can then be checked against social processes at work as revealed by textual and archaeological data. As we will see later in this unit, there existed an astronomical basis to the organization of Rgvedic itself; this helps us see Vedic ritual in a new light. That astronomy could be used for fixing the chronology of certain events in the Vedic books was shown more than a hundred years ago by Tilak and Jacobi. This internal evidence compels the conclusion that the prehistory of the Vedic people in India goes back to the fourth millennium and earlier. On the other hand, new archaeological discoveries show continuity in the Indian tradition going as far back as *7000* B.C.E. These are some of the elements in accord with the view that the Vedic texts and the archaeological finds relate to the same reality. Recent archaeological discoveries establish that the $Sarasvat\bar{\imath}$river dried up around *1900* B.C.E. which led to the collapse of the Harappan civilization that was principally located in the $Sarasvat\bar{\imath}$ region. Francfort has even argued that the $Drsadvat\bar{\imath}$was already dry before *2600* B.C.E. The region of the $Sarasvat\bar{\imath}$ and the $Drsadvat\bar{\imath}$ rivers, called $Brahm\bar{\imath}\,varta$, was especially sanctified and $Sarasvat\bar{\imath}$ was one of the mightiest rivers of the Rgvedic period. On the other hand, $P\tilde{a}ncavim\d{.}s'aBrahm\bar{a}\,na$ describes the disappearance of $Sarasvat\bar{\imath}$ in the sands at a distance of forty days on horseback from its source with the understanding of the drying up of $Sarasvat\bar{\imath}$ it follows that the Rgvedic hymns are generally anterior to *1900* B.C.E but if one accepts Francfort's interpretation of the data on the $Drsadvat\bar{\imath}$ then the Rgvedic period includes the period before *2600* B.C.E.

CHECK YOUR PROGRESS

Multiple Choice Question:

CYP 1. Which Indian scholar explained that the rotation of the Earth on its axis accounts for the daily rising and setting of the sun?

a. Bhaskara II
b. Brahmagupta
c. Aryabhata
d. Varahamihira

Fill in the blanks:

CYP2. The axis of the earth is so inclined that the **……….**end of the axis always points to the Pole Star or commonly known as Dhruv Tara.

6.6 NAKṢTRAS

Nakaṣtras are divisions of the zodiac, each spanning 13°20' of the ecliptic, and are used in Vedic astrology to understand an individual's characteristics and life path.

The *Ṛgveda* describes the universe to be infinite. Of the five planets itmentions Brhaspati(Jupiter) and Vena (Venus) by name. The moon'spath was divided into 27 equal parts, although the moon takes about $27\frac{1}{3}$days to complete it. Each of these parts was called a *nakaṣtra*. Specificstars or asterisms were also termed *nakṣatras*. *ŚatapathaBrāhman*relates a storyabout the *nakaṣtra* being as powerful as the sun in earlier times butthat they have lost this power to the sun. In view of this the etymology *na*+*kṣatra*, `no power,' is proposed. A favored modern etymology is *nak-kṣtra*`ruler over night.' One ancient name of astronomer is *nak-kṣtra-darśa.*

*Nakṣatra*are mentioned in the *Ṛgveda* and *Taittirīya* Samhitā specīcally mentions that they are linked to the moon's path.

The *Ṛgvedic* reference to 34 lights apparently means the sun, the moon, the five planets, and the 27 *Nakṣatra* In later literature the list of *Nakṣatra* was increased to 28. Constellations other than the *Nakṣatra* were also known; these include the *Rkas (the Bears)*, the two divine *Dogs (Canis Major and Canis Minor)*, and the *Boat (Argo Navis)*.Aitreya*Brāhman*a speaks of *Mṛga (Orion)* and *Mṛgavyādha (Sirius)*. The moon is called sūryaras′mi, one that shines by sunlight. *ŚatapathaBrāhman*a provides an overview of the broad aspects of Vedic astronomy.The sixth chapter *(kānda)* of the book provides significant clues.Speaking of creation under the aegis of the*Prāpati*(reference either to a star or to abstract time)mention is made of the emergence of*Aśva, Rāsabha,Aja and Kūrma* before the emergence of the earth.

*ViśvanāthaVidyālank̇āra*suggests that thsesshoul*Aja (Capricorn)* and *Kūrma (Cassiopeia)*. This identification is supported by etymological considerations.

CHECK YOUR PROGRESS

Multiple Choice Question:

CYP 3. The *Ṛgvedic* reference to

- **a.** 27 *Nakṣatra*
- **b.** 26*Nakṣatra*
- **c.** 25*Nakṣatra*
- **d.** None of these

Fill in the blanks:

CYP4. The moon's path was divided into…….. equal parts, although the moon takes about……. days to complete it. Each of these parts was called a ……………………………

6.7 ZODIAC SIGN

If one observes the movement of planets, it is seen that they also move in their own orbits `along with the Sun's path, but their path deflects north-south also. However the planets never proceed more than 9° either north or south of the ecliptic.

No.	Sign	Rashi	Extent	Lord of sign
1.	ARIES	MESHA	0° to 30°	MARS
2.	TAURUS	VRISHA	30° to 60°	VENUS
3.	GEMINI	MITHUNA	60° to 90°	MERCURY
4.	CANCER	KARKA	90° to 120°	MOON
5.	LEO	SIMHA	120° to 150°	SUN
6.	VIRGO	KANYA	150° to 180°	MERCURY
7.	LIBRA	TULA	180° to 210°	VENUS
8.	SCORPIO	VRISCHIKA	210° to 240°	MARS
9.	SAGITTARIUS	DHANU	240° to 270°	JUPITER
10.	CAPRICORN	MAKARA	270° to 300°	SATURN
11.	AQUARIUS	KUMBHA	300° to 330°	SATURN
12.	PISCES	MEENA	330° to 360°	JUPITER

Hence if a parallel line on either side of the ecliptic is drawn at an angular distance of about 9° then the ecliptic will come in the middle and either side will be a broad band/path way in which all planets can be located. This imaginary belt/band stretching about 9° north and 9° south of the ecliptic within which the planets and the Moon remain in course of their movement in the heavens, is known as Zodiac. In astrology we refer to this broad band of 18° instead of referring to the entire sky.

The zodiac signs in the Hindu (or Vedic) calendar are known as Rashis and include the same twelve signs as the Western zodiac: Aries (Mesha), Taurus (Vrishabha), Gemini (Mithuna), Cancer (Karka), Leo (Simha), Virgo (Kanya), Libra (Tula), Scorpio (Vrishchika), Sagittarius

(Dhanus), Capricorn (Makara), Aquarius (Kumbha), and Pisces (Meena). Unlike Western astrology's focus on the Sun sign, Vedic astrology emphasizes the Moon sign (Chandra Rashi), which is the Rashi where the Moon was positioned at the moment of birth.

Vedic mathematics is a system of calculation and logic used within Vedic astrology (Jyotiṣa) to interpret the zodiac, which has 12 signs and is divided into 360 degrees. This calculation includes the 27 Nakshatras (lunar mansions) and involves determining planetary positions using the sidereal zodiac. The number 108 is also significant in Vedic astrology and mathematics, representing the 12 signs multiplied by the 9 planets and connecting to the heart chakra and the Earth-Sun distance.

CHECK YOUR PROGRESS

Multiple Choice Question:

CYP 5. The *Ṛgvedic* reference to

a. 27 *Nakṣatra*

b. 26*Nakṣatra*

c. 25*Nakṣatra*

d. None of these

Fill in the blanks:

CYP6. Vedic mathematics is a system of calculation and logic used within Vedic astrology (Jyotiṣa) to interpret the zodiac, which has …….and is divided into………………….

6.8 NAKṣTRASAND CHRONOLOGY

Motivated by the then- current models of the movements of pre-historic peoples, it became, by the end of the nineteenth century, fashionable in Indologicalcircles to dismiss any early astronomical references in the Vedic literature. But since the publication of *Hamlet's Mill: An essay on myth and the frame of time* by Georgio de Santillana and Hertha von Dechend in 1969 it has come to be generally recognized that ancient myths encode a vast and complex body of astronomical knowledge. The cross-checks provided by the dating of some of the Indian myths provide confirmation to the explicit astronomical evidence related to the *Nakṣtras* that is spelt out below.

The earth's axis of rotation is tipped at an angle of $23\frac{1}{2}^{\circ}$ with respect to the direction of its orbital motion around the sun. This is what causes the changing seasons because the length of the day keeps on varying. The longest and the shortest days, also called summer and winter solstices, occur roughly near the 21st of June and and 21st December, respectively. The date of a solstice can be marked by noting that around this date the sun appears to linger at the same extreme at dawn. The days when the days and nights are equal are called equinoxes. The two equinoxes, vernal in spring and autumnal in fall, mark the halfway points between summer and autumn. The equinoxesoccur at the two intersections of the celestial equator and the ecliptic. Themotion of the moon is more complex since its orbit is inclined approximately$5^{\circ}$to the earth's orbit around the sun, and the earth's gravitation perturbsthe moon in its orbit. The resultant precession completes a cycle in 18.61years.

Due to the precession of the earth's polar axis the direction of the north pole with respect to the fixed background stars keeps on changing. The period of this precession is roughly 26,000. Polaris (α −Ursae Minoris) is the Pole star now but around 3000 B.C.E. it was α −Draconis which was followed later by β −Ursae Minoris; in C.E. 14000 it will be Vega. The equinoxes and the solstices also shift with respect to the background stars. The equinoxes move along the ecliptic in a direction opposite to the yearly course of the sun (Taurus to Aries to Pisces rather than Pisces to Aries to Taurus and so on).The vernal equinox marked an important day in the year. The sun's position among the constellations at the vernal equinox was an indication of the state of the precessional cycle. This constellation was noted by its heliacal rising. The equinoctial sun occupies each zodiacal constellation for about 2200 years. Around 5000 B.C.E. it was in Gemini; it has moved since into Taurus, Aries, and is now in Pisces. The sun spends about $13\frac{1}{3}$days in each *Nakṣtra*, and the precession of the equinoxes takes them across each *Nakṣtra* in about a 1000 years.

S.No.	Name of Nakshatra/ Star	Extent (Longitude)	Extent Sign/Rashi	Lord of Nakshatra/ Constellations	No. of years in Vimshottari Dasha
1.	Ashwini	0° to 13°20'	Mesha 0° to Mesha 13°20'	KETU	7
2.	Bharani	13°20' to 26°40'	Mesha 13°20' to Mesha 26°40'	VENUS	20
3.	Krittika	26°40' to 40°	Mesha 26°40' to Vrisha 10°	SUN	6
4.	Rohini	40° to 53°20'	Vrisha 10° to Vrisha 23°20'	MOON	10
5.	Mrigashira	53°20' to 66°40'	Vrisha 23°20' to Mithuna 6°40'	MARS	7

6.	Ardra	66°40' to 80°	Mithuna 6°40' to Mithuna 20°	RAHU	18
7.	Punarvasu	80° to 93°20'	Mithuna 20° to Karka 3°20'	JUPITER	16
8.	Pushya	93°20' to 106°40'	Karka 3°20' to Karka 16°40'	SATURN	19
9.	Ashlesha	106°40' to 120°	Karka 16°40' to Karka 30° or Simha 0°	MERCURY	17
				TOTAL	120
10.	Magha	120° to 133°20'	Simha 0° to Simha 13°20'	KETU	7
11.	Poorva Phalguni	133°20' to 146°40'	Simha 13°20' to Simha 26°40'	VENUS	20
12.	Uttra Phalguni	146°40' to 160°	Simha 26°40' to Kanya 10°	SUN	6
13.	Hasta	160° to 173°20'	Kanya 10° to Kanya 23°20'	MOON	10
14.	Chitra	173°20' to 186°40'	Kanya 23°20' to Tula 6°40'	MARS	7
15.	Swati	186°40' to 200°	Tula 6°40' to Tula 20°	RAHU	18
16.	Vishakha	200° to 213°20'	Tula 20° to Vishchika 3°20'	JUPITER	16
17.	Anuradha	213°20' to 226°40'	Vrishchika 3°20' to Vrishchika 16°40'	SATURN	19
18.	Jyeshtha	226°40' to 240°	Vrishchika 16°40' to Vrishchika 30° or Dhanu 0°	MERCURY	17
				TOTAL	120
19.	Moola	240° to 253°20'	Dhanu 0° to Dhanu 13°20'	KETU	7
20.	Poorvashadha	253°20' to 266°40'	Dhanu 13°20' to Dhanu 26°40'	VENUS	20
21.	Uttrashadha	266°40' to 280°	Dhanu 26°40' to Makara 10°	SUN	6
22.	Shravana	280° to 293°20'	Makara 10° to Makara 23°20'	MOON	10

23.	Dhanishtha	293°20' to 306°40'	Makara 23°20' to Kumbha 6°40'	MARS	7
24.	Shatabhisha	306°40' to 320°	Kumbha 6°40' to Kumbha 20°	RAHU	18
25.	Poorva Bhadra	320° to 333°20'	Kumbha 20° to Meena 3°20'	JUPITER	16
26.	Uttra Bhadra	333°20' to 346°40'	Meena 3°20' to Meena 16°40'	SATURN	19
27.	Revati	346°40' to 360°	Meena 16°40' to Meena 30° (or Mesha 0°)	MERCURY	17
				TOTAL	120

6.9 ECLIPSE

A solar eclipse occurs when the Moon passes between the Earth and the Sun, blocking the Sun's light and casting a shadow on Earth.

A lunar eclipse happens when the Earth passes directly between the Sun and the Moon, casting its shadow on the Moon. The key difference is the object blocking the light: the Moon blocks the Sun in a solar eclipse, while the Earth blocks the Sun's light from the Moon in a lunar eclipse. Solar eclipses are described in Vedic astronomy. In the context of Vedic Mathematics itself, which focuses on mathematical techniques for calculation and computation. Ancient Indian texts like the Rig Veda contain references to astronomical phenomena, including solar eclipses, which were predictable through advanced calculations involving concepts like Rahu and Ketu (the lunar nodes). These calculations allowed for the prediction of eclipses, demonstrating a sophisticated understanding of astronomy long before the concept of Vedic Mathematics was formalized. Indian astronomers, notably Aryabhata, scientifically explained that a lunar eclipse occurs when the Moon passes into the shadow cast by the Earth. These astronomers developed computational methods and calculations to determine the size of the Earth's shadow and the extent of the eclipsed part of the Moon.

CHECK YOUR PROGRESS

True\False:

CYQ7. A solar eclipse occurs when the Sun passes between the Earth and the Moon, blocking the Sun's light and casting a shadow on Earth. True\False.

CYQ8. A lunar eclipse happens when the Earth passes directly between the Sun and the Moon, casting its shadow on the Moon. True\False.

6.10 PANCHANGA

The Panchanga provides the five elements of time calculation, including tithi (lunar day), nakshatra (lunar mansion), yoga, karana, and day (varam). which are derived from astronomical calculations based on planetary motions, including the Sun and Moon, and are used to determine auspicious times for various activities. While Vedic mathematics involves complex calculations, Panchang focuses on the astrological interpretation of these astronomical events

Components of the Panchang:

The five parts of the Panchang are:

- **Vaar (Day):** The weekday, determined by the planetary lordship of the day.
- **Tithi:** The lunar day, which is one-thirtieth of the lunar month, calculated based on the angular distance between the Moon and the Sun.
- **Nakshatra:** The 27 lunar mansions or constellations through which the Moon passes.
- **Yoga:** A specific planetary combination that occurs at a particular time, influencing the nature of the day.
- **Karana:** A calculation that is half of a Tithi and is used for certain religious and auspicious timings, such as marriages.

Important points:

- The number nine is significant, and contributes to the elegant mathematical symmetry of Jyotish: 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = 108.
- 9 planets x 12 signs or houses = 108, sacred number of cosmic wholeness in India, and sacred number to many other traditions as well.
- The distance between the Sun and the earth and the Moon and the earth is 108 x their respective diameters.
- The diameter of the Sun is 108 x the diameter of the earth.
  There are 27 nakshatras, lunar constellations, and each planet rules three of them: 9 x 3 = 27.
- The 27 nakshatras spread over the 4 elements: 4 x 27 = 108.
- The Ninth Division or Navamsa is the most important divisional chart, and each sign of 30° divides equally into 3° 20' to form it: 9 x 3° 20' = 30°.
- In our Hindu-Arabic numeral system, 9 is the total number of basic digits. The sum of the digits of 108 is 9: 1 + 0 + 8 = 9.
- 108 is divisible by the sum of its digits:108 ÷ 9 = 12.
  Indeed, the nine planets travelling through the twelve signs comprise the whole of manifest existence! The Vedic system forms a complete mirror of reality.

6.11 EQUINOX

We all know about two motions of the Earth, Rotation and Revolution. There are many other motions of the Earth and one of them is the Precession of Equinox. This 3rd motion of the Earth resembles Earth's movement like a Top where the axis of the Earth keeps moving in a Circular shape. The concept of Precession of Equinox is a common concept in the world of Astronomy (Jyotisha/Khagolshastra in the Bharatiya culture). Let us understand the concept. Any point on the equator (specifically the intersection of Prime Meridian at the time of day of Spring (20/21-March in current time 2024 CE) or Autumn (21/22-September in current time 2024 CE) Equinox when projected on the celestial equator coinciding with the Ecliptic (plane of earth's revolution), is called the point of Equinox. On the day of Equinox, day and night are of equal length on the Equator. It is believed that this point keeps on moving on the ecliptic in an westward direction with a very slow motion due to the forces applied by the Sun, Moon and Jupiter on the Earth, giving birth to the concept of Precession of Equinox. We all know that the Earth's axis which is an imaginary line between the Earth's North and South Pole is tilted by an angle of 23.44° in

current time (2024 CE) with respect to the perpendicular line to the ecliptic (plane of Earth's revolution). The axis points to the North Celestial Pole (Pole Star above the North Pole) in the Northern direction and towards the South Celestial Pole in the Southern direction. Due to the continuous movement of the point of Equinox over the years slowly westwards, the axis also rotates and makes a circle and returns to the point of origin of the circle after a certain number of years. The circular path made by the axis is called the Circle of Precession.

While the axis traverses on the Circle of Precession, it keeps on pointing to the new Pole Star as years pass by and comes back to the one pointing where it started from. The axis points to the Pole Star named Polaris in the current time (2024 CE).

The axis pointed to a pole star called Vega thousands of years back which is considered approximately halfway on the Circle of Precession. This phenomenon of movement of the point of Equinox across the ecliptic in an westward direction thereby making the axis also follow a circular path along the **Circle of Presession** is called Precession of Equinox. As we understood that the axis moves in a counterclockwise movement due to the movement of the point of Equinox, the backdrop stars for the Equinox point also keep on changing as it moves along the circle of precession.

These backdrop stars are 27 Nakshatras which are equally spaced across the ecliptic at **13.33 degrees** apart as stated in the formula ***[360 degrees / 27 = 13.33 degrees]***. As the point of Equinox covers **13.33°**, the full moon in that lunar month points to a Nakshatra next and westwards the earlier one. 12 out of 27 Nakshatras are used for marking the lunar months. As the point of Equinox traverses 26.67° i.e. ***[2 x 13.33 degrees = 26.67 degrees (2 Nakshatra Stars)]***, this changes the lunar month coinciding with the month of the Gregorian calendar. For example, if Equinox at one point in time falls on the lunar month of Chaitra and the Gregorian month of March in year X, then after a couple of years, in year Y, the lunar month of Chaitra will fall in the Gregorian month of April. The effect of the Precession of Equinox also impacts Seasons coinciding with Bharatiya (Indian) Lunar and Gregorian months. There are 6 seasons in Bhartiya tradition, separated with each one covering ~2.25 Lunar or Gregorian months.

For example, if the point of Vernal Equinox (referred to as VE hereafter) shifts 26.67 i.e. 2

Nakshatras, then it's called a shift of 1 lunar month, but the season still remains the same. But if VE shifts by 53.33 i.e. 4 Nakshatras it then marks a shift of ~2.25 lunar months and 1 season. This implies that the Lunar month of Chaitra falls in the Vasant/Spring season then it will start falling in the Greeshma (Summer) season after the VE shifts by 53.33 westwards.

6.12 SUMMARY

This unit is presentation of the concept Akash Ganga, Solar system: Sun, Earth, Moon, Diameter of the Sun and Diameter of the Earth and Moon. Simultaneously we have presented the concept of Evolution of the Vedic thought, Nakaṣtras, Zodiac Sign, Nakaṣtras and the chronology and eclipse. This unit explain importance of Vedic Mathematics into astronomy. After writing the presented unit we understands that mathematics was used a lot in the Vedic period.

6.13 GLOSSARY

i. Akash Ganga
ii. Solar system
iii. Sun
iv. Earth
v. Moon
vi. Diameter of the Sun
vii. Diameter of the Earth and Moon
viii. Evolution of the Vedic thought
ix. Nakaṣtras
x. Zodiac Sign
xi. Nakaṣtras and the chronology
xii. Eclipse
xiii. Equinox

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6.15 SUGGESTED READINGS

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6.16 TERMINAL QUESTIONS

TQ1. Define Akash Ganga……………………………………………….
TQ2. Describe Solar system…………………………………………………….
TQ3. Define Sun, Earth and Moon……………………………………………….
TQ4. Explain the Diameter of the Sun………………………………………
TQ5. Describe the Diameter of the Earth and Moon………………………..
TQ6. Describe the Evolution of the Vedic thought………………………….
TQ7. Define Nakaṣtras and Zodiac Sign……………………….
TQ8. Define the eclipse………………………………………………..

6.17 ANSWERS

CYP1. (c) Aryabhata

CYP2. Northern

CYP3. 27 *Nakṣatra*

CYP4.27,27 $\frac{1}{3}$,*nakaṣtra*

CYP5.27 *Nakṣatra*

CYP6. 12 signs and 360 degrees

CYP7. False

CYP8. True

BLOCK -III
VEDIC MATHEMATICS: ARITHMETIC CALCULATION

UNIT 7: BRIEF INTRODUCTION OF VEDIC ARITHMETIC –I

CONTENTS:

7.1 INTRODUCTION

Vedic Mathematics is an ancient and unique system of mathematical techniques that originated in India. It offers a faster, more efficient, and more creative way of solving mathematical problems. The system is based on 16 Sutra**s** (formulas) and **13** Sub-Sutras **(**sub-formulas), derived from ancient Indian scriptures called the Vedas—especially the Atharva Veda**.**

This system was rediscovered and compiled by Jagadguru Swami Bharati Krishna Tirthaji Maharajin the early 20th century, and later published in his book *Vedic Mathematics* in 1965.

The word *Vedic* is derived from the Sanskrit word *Veda*, which means knowledge**.** Although these methods are believed to have existed since the Vedic period they were lost over the centuries. Swami Bharati Krishna Tirthaji, a scholar in Sanskrit and Mathematics, studied the ancient texts and reconstructed the system through instant meditation and research between 1911 and 1918.

7.2 OBJECTIVES

After studying this unit learner will be able

1. To reduce Fear and Anxiety of Mathematics
2. To make Mathematics Easy and Enjoyable
3. To simplify Complex Calculations
4. To develop Speed and Accuracy
5. To encourage Mental Calculation

7.3 ADDITION AND SUBSTRACTION

Vedic Mathematics provide various techniques for addition and subtraction, making calculations faster and more efficient. Some key methods include:

Addition Techniques:

(i) Ekadhikena Purvena (By One More than the Previous): This technique involves adding numbers by grouping and using mental calculations.

Example: Addition: 457 + 279 =?

Using Ekadhikena Purvena, we can add group numbers and calculate: 457 + 200 = 657, 657 + 70 = 727, 727 + 9 = 736

(ii) Nikhilam Navatashcaramam Dashatah (All from 9 and Last from 10): This method is used for subtracting numbers close to the base (10, 100, 1000, etc.).

Subtraction Techniques:

(i) Nikhilam Navatashcaramam Dashatah: This method is also used for subtraction, where numbers are subtracted from the base.

This sutra is used when subtracting from powers of 10 (like 1000, 100, etc.)

Example: 1000 – 743
→ Subtract each digit of 743 from 9 except the last (from 10):
→ 9 - 7 = 2, 9 - 4 = 5, 10 - 3 = 7
→ Answer = **257**

Example: Subtraction: 1000 - 457 = ?

Using Nikhilam Navatashcaramam Dashatah, we can subtract: 1000 - 457 = 543

(ii) Ekanyunena Purvena (By One Less than the Previous): This technique involves subtracting numbers by grouping and using mental calculations.

7.4 VINCULUM

In Vedic Mathematics, a Vinculum (also called a bar number) is a way of expressing numbers using negative digits**.** It is represented by placing a horizontal bar above the number This method helps to simplify calculations by reducing the need for carrying and by making certain arithmetic operations easier and faster to perform.

Example

Let us take the number 867

Instead of using

8 → keep as it is

6 → write as **(10 - 4) = -4** with carry

7 → write as **(10 - 3) = -3** with carry

Using Vinculum, it becomes: $\overline{\mathbf{843}}$, here, $\overline{\mathbf{4}}$ means -4 and $\overline{\mathbf{3}}$ means -3 So, 867 is rewritten as: because $\overline{\mathbf{943}}$ → 9 + 1 = 10 → -1 carried forward, and so on. This simplifies arithmetic operations like addition, subtraction, multiplication, especially using other Vedic sutras like Nikhilam or Urdhva Tiryak**.**

<u>Vinculum Chart (Bar Numbers and Their Positive Equivalents):</u>

Digit	Vinculum form	Equivalence value
1	$\overline{9}$	10-1=9
2	$\overline{8}$	10-2=8
3	$\overline{7}$	10-3=7
4	$\overline{6}$	10-4=6
5	$\overline{5}$	10-5=5
6	$\overline{4}$	10-6=4
7	$\overline{3}$	10-7=3
8	$\overline{2}$	10-8=2
9	$\overline{1}$	10-9=1

Note: A bar over a digit means you subtract it from 10 and carry 1 to the next digit**.**

Example: Convert 867 to Vinculum Form.

Solution: First We want to simplify large digits using bar numbers.

8→10 - 2 = $\bar{2}$ (carry 1)

6→10 - 4 = $\bar{4}$ (carry 1)

7 →10 - 3 = $\bar{3}$ (carry 1)

Now apply carries

8 + 1 = 9

6 + 1 = 7 → 10 - 3 = $\bar{3}$

7 + 1 = 8 → 10 - 2 = $\bar{2}$

So, 862 = $\overline{\mathbf{932}}$

7.5 BEEJANK (REMAINDER BY NINE)

Beejank is a Sanskrit word which meaning means**"**seed Number"**.** In Vedic Mathematics, it refers to the remainder obtained when a number is divided by 9. It is also known as reducing a number to a single digit using the digital root or remainder method.

How to find a Beejank of a Number

To find the Beejank (remainder by 9), add all the digits of the number repeatedly until you get a single-digit number.

First add the digits of the number. If the result is a two-digit number, add those digits again**.** Then Continue until a single-digit number is obtained. That single digit is the Beejank **or** remainder when divided by 9**.**

Example1: Find the Beejank of 587.

Solution: First we will add

5+8+7 =20

2+0 =0, Beejank =2

Example 2: Find the Beejank of 1437:

Solution: First we will add

1+4+3+7 =15

1+5=6, Beejank =6

Example: Find the Beejank of 999.

Solution: First we will add

9+9+9 = 27

2+7= 9, Beejank =9

(**Note**: If the sum is a multiple of 9, the Beejank is 9, not 0)

Number	Step I add digit	Step II add again (if Neded)	Beejank
236	2+3+6 =11	1+1=2	2
579	5+7+9=21	2+1=3	3
862	8+6+2=16	1+6=7	7
269	2+6+9=17	1+7=8	8
3897	3+8+9+7= 27	2+7=9	9

***Table: 7.5.1** Table for understanding Beejank (Remainder by Nine):*

Operation	Expression	Beejank of LHS	Beejank of RHS	Is it Correct
Addition	123 + 456 = 579	6 + 6 = 3	5 + 7 + 9 = 21 → 3	Yes
Subtraction	845 - 312 = 533	(8+4+5)-(3+1+2)=17-6=11→2	5+3+3 = 11 → 2	Yes
Multiplication	23 × 14 = 322	2+3 = 5, 1+4 = 5 → 5×5 = 25 → 7	3+2+2 = 7	Yes
Complex Expression	321 + 123 = 444	3 + 6 = 9 → 9×2 = 18 → 9	1+1+5+8 = 15 → 6	No

***Table 7.5.2 Table** for Mixed Operations and Beejank Check*

Rule for Checking Beejank

1. If Beejank of LHS = Beejank of RHS, the result is likely correct**.**

2. If they don't match, the result is definitely incorrect**.**

Note: Beejank is a quick-check tool**,** not a substitute for exact calculations.

7.6 KAPAREKARCONSTANT

The Kaprekar Constant **is** 6174, discovered by the Indian mathematician **D. R. Kaprekar**. It has a fascinating property associated with a special four-digit number process. Although the Kaprekar Constant (6174) is not found directly found in ancient Vedic Mathematics, it matches beautifully with the spirit and philosophy of Vedic Math—simplicity, patterns, mental calculation, and number magic**.** Here are some points where we explained. How Kaprekar Constant (6174) is connected with Vedic maths.

(i) Pattern Recognition: This is the process reveals a hidden numerical patternthat always leads to 6174.

(ii)Number Operations: Vedic mathematics often simplifies operations through manipulation of numbers**,** just as here the numbers are rearranging.

(iii) Iterative Reasoning: Many Vedic sutras encourage step-wise simplification**,** similar to Kaprekar's repeated subtraction.

(iv)Magic Numbers: Vedic math studies special numbers like 9, 108, etc. Kaprekar's 6174 can be studied from a similar mystical perspective.

Rule for finding Kaprekar Constant (6174)

(i) Take any4-digit number, using at least two different digits (e.g., 3524).
(ii) Arrange its digitsindescending and ascendingorder.
(iii) Subtract the smaller number from the bigger one.
(iv) Repeat the process with the result.

You will always reach 6174 in at most 7 steps**.**

Example1: Find Kaprekar Constant (6174) of 7429.

Solution: Here we explained how to find **Kaprekar Constant (6174)** with the help of table.

Steps	Descending	Ascending	Subtraction	Result
1	9742	2479	9742-2479	7263
2	7632	2367	7632-2367	5265
3	6552	2556	6552-2556	3996
4	9963	3699	9963-3699	6264
5	6642	2466	6642-2466	4176
6	7641	1467	7641-1467	6174

Note: It Works only with 4-digit numbers**.**6174 is called the Kaprekar constant**.** If you start with any number and apply this process, you will end up at 6174 (unless all digits are the same like 1111, 2222, etc.).

7.7 MULTIPLICATION (NIKHILAM SUTRA)

The **Nikhilam Sutra** is one of the fundamental sutras (formulas) in Vedic Mathematics. It is used for quick multiplication, especially when numbers are close to a power of 10 like 10, 100, 1000, etc. It works well for both numbers less than and greater than the base. It involves finding the difference of the numbers from the base, using cross- addition or subtraction with the deficiencies, and then adding those results. It means

"Nikhilam Navatashcaramam Dashatah"

(All from 9 and the last from 10)

Here we are given both caeses:

Case 1: Both Numbers Less Than Base (e.g., 100)
Example: **97 × 98**
Base = 100
97 is 3 less than 100 → (-3)
98 is 2 less than 100 → (-2)
Now,
Left part:97 – 2 = 95 or 98 – 3 = 95
Right part: (-3) × (-2) = 6
Answer: 9506

Case 2: Both Numbers More Than Base (e.g. 100)

Example: 103 × 104
Base =100
103 is 3 more than 100 → (+3)

104 is 4 more than 100 → (+4)
Left part: 103 + 4 = **107** or 104 + 3 = 107
Right part: 3 × 4 = **12**
Final Answer = 10712

Case 3: One Number More, One Less

Example: 96 × 103

Base = 100

96 is –4 from 100

103 is +3 from 100

Left part: 96 + 3 = 99 (or 103 – 4)

Right part: (–4) × (+3) = –12

Since right part is negative, borrow 1 from left(i.e., 99 – 1 = 98)

Now, 100 – 12 = 88

Final Answer = 9888

With the help of Nikhilam Sutrawe can speeds up multiplication, avoid long multiplication steps and Great for mental math.

7.8 DIVISION OF A NUMBER

In Vedic Mathematics, division is simplified using sutras (formulas) that make calculations faster and more intuitive, especially for mental math. The main sutras used for division are:

Parāvartya Yojayet:

"Transpose and Adjust"

This sutra is used when the divisor is close to powers of 10 (like 10, 100, 1000).

Method 1: Parāvartya Yojayet (Division by Nikhilam)

Best used when divisor is near base (10, 100, etc.)

Example: 1225 ÷ 99

Solution: Step 1: Base = 100 → Deficiency of divisor (99) = 1
Step 2: Write the dividend: 12 | 25 (divide as per base digits, here 2 digits)
Step 3: Multiply left side (12) by deficiency (1) → 12
Step 4: Subtract: 25 – 12 = 13

Quotient = 12, Remainder = 13

Method 2: Straight Division using Base Multipliers (flag Method)

Used when divisor is **slightly more than the base** (like 101, 1001). Here, we use the excess over the base.

Example: 1025 ÷ 101

Solution: Step 1: Base = 100 → Surplus = +1
Step 2: Split dividend: 10 | 25
Step 3: Multiply quotient part (10) by +1 = 10
Step 4: Add: 25 + 10 = 35

Quotient = 10, Remainder = 35

Method 3: Normal Long Division using Vinculum and Vedic Shortcuts

Example: Divide 432 by 12

Step 1: Set up the division problem

Dividend: 432

Divisor: 12

Step 2: Apply Vedic shortcut

Using the Vedic method, we can simplify the division by breaking down the divisor into smaller factors. In this case, 12 can be factored as 4 × 3.

Step 3: Perform the division

First, divide 432 by 4:

432 ÷ 4 = 108

Then, divide 108 by 3:

108 ÷ 3 = 36

Step 4: Write the final answer

Quotient: 36, Remainder: 0

CHECK YOUR PROGRESS

MULTIPLE CHOICE QUESTIONS

1. **Which sutra is used for multiplication of numbers close to a base?**
   A. Ekadhikena Purvena
   B. Nikhilam Navatashcaramam Dashatah
   C. Urdhva Tiryagbhyam
   D. Paravartya Yojayet

2. **The sutra "All from 9 and the last from 10" is useful for:**
   A. Division
   B. Addition
   C. Subtraction from Base
   D. Square roots

3. **Which is the fastest Vedic method for multiplying any two-digit numbers?**
   A. Long multiplication
   B. Column method
   C. Urdhva Tiryagbhyam
   D. Repeated addition
4. **Vedic arithmetic techniques are especially helpful in:**
   A. Programming languages
   B. Fast calculations and competitive exams
   C. Legal writing
   D. Grammar correction
5. **What does the sutra "Ekadhikena Purvena" mean?**
   A. All from 9
   B. By one more than the previous one
   C. The product of sums
   D. Division by 10
6. **The method of solving from left to right is a key feature of:**
   A. Western math
   B. Vedic mathematics
   C. Boolean algebra
   D. Set theory

7.9 SUMMARY

This unit introduces the fundamental concepts and techniques of **Vedic Mathematics**, focusing on methods that enhance **speed, accuracy, and mental calculation skills**. It covers simplified approaches to **addition and subtraction** using complementary numbers and left-to-right methods. The unit explains the use of **Vinculum**, where negative digits are represented to make calculations easier. Learner's learn about **Beejank (digital root)** as a tool for verifying calculations and understanding number properties. Mental techniques for **multiplication tables** are introduced through Vedic sutras such as **Ekādhikena Pūrvena** and **Nikhilam Navataścaramam Daśataḥ**. The unit also emphasizes handling **mixed operations** efficiently using Vedic shortcuts. Additionally, the unit explores number patterns through the **Kaprekar Constant (6174)**, demonstrating logical and systematic thinking. Overall, this unit builds a strong foundation in Vedic Mathematics by developing numerical reasoning, mental agility, and confidence in problem-solving.

7.10 GLOSSARY

- **i. Vedic Mathematics:** An ancient Indian system of mathematics based on 16 sutras (aphorisms) from the Vedas.
- **ii. Sutra:**A short, formula-like statement that conveys a mathematical method or rule.
- **iii. Sub-Sutra:** A subsidiary or supporting rule that helps in specific cases related to the main sutras
- **iv. Nikhilam Sutra:**“All from 9 and the last from 10” – used in subtraction and multiplication near base values.
- **v. Ekadhikena Purvena :** “By one more than the previous one” – useful for special division and squaring numbers ending in 9.
- **vi. Base method:**A technique that simplifies calculations by using powers of 10 (10, 100, 1000, etc.).
- **vii. Speed Mathematics:**Fast methods of calculation developed through Vedic techniques.
- **viii. Mental Math:** Performing calculations mentally without writing or using a calculator.

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7.12 SUGGESTED READING

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2. "Vedic Mathematics Made Easy" by Dhaval Bathia: A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.

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4. **"Mathematics in India" by Kim Plofker**: A scholarly account of the historical development of mathematics in India.Provides context on how Vedic methods evolved over time.

5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

7.13 TERMINAL QUESTIONS

1. How does the Vedic method of left-to-right addition differ from the conventional right-to-left method?

....

....

2. What is the Beejank of 738? How can it be used to check the correctness of an arithmetic operation?

....

....

3. Multiply 98 × 97 using the Nikhilam Sutra.

....

....

4. Show the Kaprekar process starting from the number 3524. How many steps does it take to reach 6174?

……
……

5. Use Nikhilam method to multiply 104 × 106.

……

6. Explain how the "Purvena" principle simplifies certain multiplication patterns.

……

7. Convert the number 9876 into its vinculum form.

……

8. Find the square of 45 using base 50 or base 40 technique.

……

7.14 ANSWERS

MCQ1.B
MCQ2. C
MCQ3.C
MCQ4.B
MCQ5.B
MCQ6.B

UNIT 8: TECHNIQUES OF ROOT FINDING IN VEDIC MATHEMATICS

CONTENTS:

8.1 INTRODUCTION

Root finding is an important process in mathematics, under which the square root, cube root or higher root of a number is found. In general mathematics, this process is considered long and complicated, but in Vedic mathematics, many simple, quick and logical methods of finding the root are described.Through various formulas of Vedic mathematics, square root and cube root can be calculated without any difficult calculations, just mentally or with little effort. For example, **Ekādhikena Pūrvena** method is helpful in calculating the square and square root.

"Ekanyūnena Pūrvena method is also helpful in finding the square and root of numbers.The use of these techniques not only helps in faster calculation but also develops computational skills, logical thinking and confidence in learners.

In this chapter, we will study the techniques of finding square roots and cube roots through the major methods of Vedic mathematics and see how these methods demonstrate the amazing simplicity and efficiency of ancient Indian mathematics.

8.2 OBJECTIVES

After studying this unit learner will be able

1. To understand the concept of root.
2. To know the traditional methods of finding square root and cube root.
3. To learn to find square root and cube root by simple techniques of Vedic Mathematics.
4. To do faster calculations using “Ekadhiken Puryvena” and “EkanyūnenaPuryena” methods.
5. To develop mental calculation ability and confidence.
6. To make calculations more accurate, quick and interesting.
7. To understand the utility of Vedic Mathematics in solving modern mathematical problems.

8.3 FINDING SQUARE OF A NUMBER

Vedic mathematics provides a collection of powerful techniques for squaring numbers, using formulas specifically designed for various numerically designed for various numerical patterns. These methos allow for fast and accurate mental calculations, making squaring easier. Here we are explained some Vedic mathematics techniques for finding square of any number.

(i) Nikhilam Sutras (Near Base Method)

This method is used when the number is close to a power of 10 like 10, 100,1000 etc.\

Sutra: ***“Nikhilam Navatashcaramam Dashatah”*** – All from 9 and the last from 10.

Example 1: Find the square of 98.

Solution: Step 1: Base = 100 → Deficiency = 100 – 98 = 2
Step 2: Left part = 98 – 2 = 96
Step 3: Right part = 2^2 = 04
Answer: 9604

Example 2: Find the square of 99.

Solution: Step 1: Base =100 → Deficiency = 100 – 99= 1
Step 2: Left part = 99-1= 98
Step 3: Right part = 1^2 = 01
Answer: 9801

(i) Urdhva Tiryagbhyam Sutra (Vertically and Crosswise Method):
This is the general multiplication formula and can be used to square a number of two or more digits.

Example: Find the square of **32**
Step 1:
Square of 2 = 4 → Write 4
Double of (3×2) = 12
Square of 3 = 9
Now arrange:
→ Write 9 (units), carry 1 from 12 → Write 3 + carry 1 = 4
→ Write 4 (hundreds)
Answer: 529

8.4 FINDING CUBE OF A NUMBER

In Vedic Mathematics**,** finding the cube of a number can be simplified by using techniques inspired by a special algebraic identity and sutras such **as** Urdhva Tiryagbhyam and Anurupyena**.**

Method 1: Nikhilam Sutra (Cube near base): For numbers close to a base (like 10, 100, etc.), use:

$(a-b)^3 = a^3 - b^3 - 3ab(a\text{-}b)$

Example: Find cube of98.

Solution: Let x = 100**,** y = 2

$98^3 = (100-2)^3 = 100^3 - 3 \times 100 \times 2(100-2) - 2^3$

$= 100^3 - 3 \times 100^2 \times 2 + 3 \times 2^2 \times 100 - 2^3$

=1000000- 60000 +1200-8 = 940192

8.5 SQUARE ROOT

Vedic Mathematics provides an efficient method for finding square roots using the "Vilokanam" method or "By Inspection" method. This technique involves estimating the square root by analysing the last digit of the number.

Steps to Find Square Root using Vedic Method:

(i) Identify the last digit: Look at the last digit of the number. This will help you determine the possible last digit of the square root.

(ii) Determine the range: Find the two perfect squares between which the number lies.

(iii) Estimate the square root: Use the Vilokanam method to estimate the square root based on the last digit and the range.

Example: Find the Square Root of 169

(i) Last digit: The last digit is 9. Possible last digits of the square root are 3 or 7.

(ii) Range: 169 lies between 144 (12^2) and 169 (13^2).

(iii) Estimate: Based on the last digit and range, the square root is likely 13.

Example: Finding square root of 2025.

Solution: Step 1: Divide the number into two-digit groups from right to left: 2025 → $\frac{20}{25}$

Step 2: Find the square root of the left group (20):

Find the largest number whose square is ≤ 20.

4^2 = 16 and 5^2 =25 >20⇒Take 4

Step 3: Last digit of square root
Look at the last digit of the original number (2025 ends in **5**).
Only numbers ending in **5** give squares ending in 25. So the last digit is **5**.

Step 4: Combine: $\sqrt{2024}$ =45

8.6 CUBE ROOT ROOT

In Vedic Mathematics**,** cube roots of perfect cubes can be found quickly using simple mental techniques — especially for 3-digit, 4-digit, or 5-digit perfect cubes**.**

Example: Find cube root of 2197.

Step1: Take the last 3 digit as one group and rest as another.

$2197 \rightarrow \frac{2}{197}$

Left group: 2→ Closest cube ≤ 2 is $\mathbf{1^3 = 1}$

→ So, first digit =1

Step 2: see the last digit (of original number)

7 is last digit of 2197, Now finds, which cube ends in 7?

The number is 3^3 =27
→ So last digit =3
So that the final answer of $\sqrt[3]{2197}$ is 13.
Example 2: Find cube root of $\sqrt[3]{13824}$ is

Take the last 3 digit as one group and rest as another.

$13824 \rightarrow \frac{13}{824}$

Left group: 13→ Closest cube ≤ 13 is 2^3 **= 8**

→ So, first digit =2

Step 2: see the last digit (of original number)

4 is last digit of 13824, Now finds, which cube ends in 4?

The number is 4^3 =64
→ So last digit =4
So that the final answer of $\sqrt[3]{13824}$is 24.

8.7 DIVISIBILITY

Vedic Mathematics provide elegant and efficient methods for testing divisibility of numbers, often much faster than traditional long division.

Rules for divisibility

(i) Double the last digit.
(ii) Subtract the result from the rest of the number.
(iii) If the result is divisible by 7, the original number is divisible.

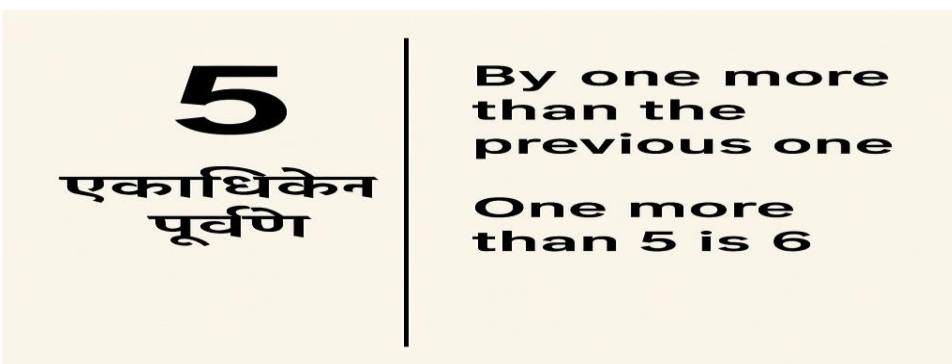
Example: Divisibility by 7 with the help of Vedic mathematics.
Solution: Suppose example: 203
→ Double last digit: $3 \times 2 = 6$
→ $20 - 6 = 14$ → 14 is divisible by 7 → so 203 is divisible by 7.

Example: Divisibility by 11 (Beejank Method).
Solution: Alternately subtract and add the digits. If result is 0 or divisible by 11, then it is divisible.

Example: 121 → $1 - 2 + 1 = 0$ → divisible by 11.

8.8 EKADHIKEN PURVEN METHOD

The Meaning ofEkadhikena Purvena Method is "By one more than the previous one".This principle of incrementing a number by one is a foundational concept in many Vedic Mathematics techniques. "This is one of the key sutras in Vedic Mathematics Which is primarily used for Finding the squares of numbers ending in 5 and converting fraction with denominators ending in 9 to recurring decimals.



Ref: www.google.com Fig 8.1

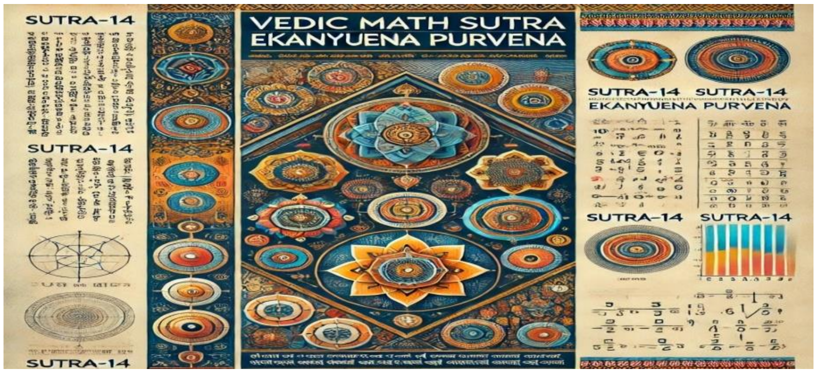
8.9 *EKANYUNEN PURVEN METHOD*

Ekanyunena Purvena is a Sanskrit sutra from **Vedic Mathematics**, and it translates to:

"Ekanyunena Purvena" = "By one less than the previous"

This sutra is used mainly for multiplication of numbers close to a base like 10, 100, 1000, etc. It works especially well when one number is just below a power of 10**,** such as 99 (which is 1 less than 100), 999 (1 less than 1000), and so on.

Ekanyunena Purvena – "Subtract 1 from the other number (the purva), and subtract the result from 10, 100, 1000 accordingly."



Ref:inquiry@moonpreneur.com Fig 8.2.

Example 1: Multiply 89× 88.

Solution: Here, 99 = 100 – 1
Apply **Ekanyunena Purvena**:

(i) First, we will Take the first number which is (89)
(ii) Then Subtract 1 from it→ 89 − 1 = 88
(iii)Now Take complement of 89 from 100 → 100 − 89 = 11
(iv) Now write:
Left side = 88, Right side = 11
(v) Final answer = 8811

So, 89 × 99 = 8811

Example 2: Multiply 997 × 999

Solution: Here 999 = 1000 – 1

Ekanyunena Purvena:

First, we will Take the first number which is (997)

(i) Then Subtract 1 from it → 997 − 1 = 996
(ii) Now Take complement of 997 from 1000 → 1000 − 997= 003 (write in 3 digit)
(iii) Now write:
Left side = 996, Right side = 003
(iv) Final answer = 996003

So 997× 999 = 996003

CHECK YOUR PROGRESS

MULTIPLE CHOICE QUESTIONS

1. What is the correct meaning of square root?
a) Dividing a number by two
b) The number obtained by multiplying a number by itself
c) Multiplying a number by 10
d) Dividing a number by two

2. Techniques of Vedic Mathematics are famous for what?
a) Complex calculations
b) Slow method
c) Simplex and Quick calculation
d) Only geometry

3. What does the sutra "Ekadhikena Purvena" mean?
a) All from 9
b) By one more than the previous one
c) The product of sums
d) Division by 10

4. The method of solving from left to right is a key feature of:
a) Western math
b) Vedic mathematics
c) Boolean algebra
d) Set theory

8.10 SUMMARY

In this unit, learners study the **Ekādhikena Pūrvena** method, which means **"by one more than the previous one."** This Vedic Mathematics technique helps simplify calculations by using the value of the **previous digit or number**. It is especially useful for **division problems**, numbers **ending in 9**, and for understanding methods related to **square roots**. The unit also introduces the **Pūrvena Method**, which extends this idea by using earlier results to solve new steps in a calculation. These methods help learners perform **arithmetic and algebraic calculations more quickly and accurately**, while improving mental math skills and logical thinking.

8.11 GLOSSARY

i. **Root:**The value of a number which when multiplied by itself repeatedly gives the original number.
ii. **Square Root:** Second root of a number.
iii. **Cube Root:**Third root of a number.
iv. **Sutra**: Simple rule or method used in Vedic mathematics.
v. **Nikhilam Sutra:**"All from 9 and the last from 10" – used in subtraction and multiplication near base values.
vi. **Ekadhikena Purvena :**"By one more than the previous one" – useful for special division and squaring numbers ending in 9.
vii. **Mental Calculation:**Calculation done in the mind without using pen and paper.

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5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

8.14 TERMINAL QUESTIONS

1. Describe the "Ekadhiken Poorven" method in Vedic mathematics and explain it with an example……………………………………………………………………………………….

3. Explain the importance of the "Ekanunen Poorven" formula and show how to find the square of a number using it……………………………………………………………………………….

3. What is the difference between the traditional and Vedic method of finding the square root? Write in detail…………………………………………………………………..

4. What is the role of Vedic mathematics in mental calculation? Explain in detail………………….

5. Use Nikhilam method to multiply 104×106……………………………………….

6. Explain how the "Purvena" principle simplifies certain multiplication patterns……………..

7. Find the square of 45 using base 50 or base 40 technique…………….

8.15 ANSWERS

MCQ1. B
MCQ 2.C
MCQ3.B
MCQ4.B

UNIT 9: ALGEBRAIC METHODS IN VEDIC TRADITION

CONTENTS:

9.1 INTRODUCTION

The Vedic tradition offers a remarkable and insightful approach to algebra, blending intuitive reasoning with elegant computational techniques. Long before modern algebraic symbolism emerged, ancient Indian scholars used geometric interpretations, numerical patterns, and sutra-based reasoning to solve algebraic problems with speed and clarity. Their methods emphasized simplicity, mental processing, and the discovery of hidden relationships among numbers. This unit**,** Algebraic Methods in Vedic Tradition, introduces the fundamental principles and Vedic sutras that form the basis of algebraic manipulation in ancient Indian mathematics. Sutras such as *Ānurūpyena* (proportion), *ParāvartyaYojayet* (transposition and adjustment), *Sankalana-Vyavakalanabhyām* (addition and subtraction), and *SunyamSamyasamuccaye* (when the sum is the same, that sum is zero) provide powerful tools for solving equations, factoring expressions, and handling algebraic identities. The Vedic approach does not merely offer shortcuts; it cultivates a deeper understanding of algebraic structure. By recognizing numerical symmetry, cancellation patterns, and proportional logic, learners learn to solve linear and quadratic equations, factor polynomials, and simplify expressions mentally, without depending solely on lengthy procedural steps. These methods enhance problem-solving speed and build confidence in manipulating algebraic forms—skills valuable in higher mathematics and competitive examinations. In this unit we define Divisibility rule. Modular arithmetic. Diophantine equation. Multiplication, (Urdhva-

Tirayak Sutra). Mountain Expansion. Power. Least common multiple. Recurring decimal. Solution of Equations and Partial Fraction.

9.2 OBJECTIVES

After studying this unit learners will able

1. To expand the foundational skills gained in Vedic Arithmetic – I through advanced techniques.
2. To understand and use divisibility rules and digital root methods for verification and simplification of calculations.
3. To develop speed, accuracy, and mental agility in solving a wide range of arithmetic problems.

9.3 VEDIC METHODS TO PRIME FACTORIZATION

In **Vedic Mathematics**, prime factorization can be understood using mental calculation techniques, divisibility rules, and sutras (formulas)such as "Ekadhikena Purvena", "Nikhilam Navatashcaramam Dashatah", and"Paravartya Yojayet." These methods simplify the process by quickly identifying divisibility, and often turns complex steps into efficient mental shortcuts. Here we are giving some examples of Vedic Methods to Prime factorization.

(i) Testing divisibility using prime roots (Beejank method),
(ii) Using Nikhilam Sutra **t**o quickly divide numbers near base powers (like 10, 100),
(iii) Using Sutra-based factor factorization instead of traditional long division.

Steps	Vedic Method/Rule	Explanation
1. Identify divisibility	Beejank (Digital Sum)	Add digits. If sum divisible by prime → original number is divisible
2. Apply divisibility rules	Vedic divisibility rules	Use shortcuts to test divisibility by 2, 3, 5, 7, 11, 13, etc.
3. Divide using Sutras	Nikhilam Sutra / ParavartyaYojayet	Quick division for numbers near base powers (10, 100, etc.)
4. Repeat factorization	Continue step-by-step with quotient	Until all factors are prime
5. Confirm with Beejank	Cross-check with digital sum logic	Rechecking your prime result with digit sum

***Table 8.3.1** Vedic Methods to Prime Factorization*

Examples: Prime factorization of 198

Solution: Step 1. Check divisibility

First, we will find Beejank of 198

Beejank = 1+0+9+8 =18, which is divisible by 9

Step 2: Divide

198÷ **9 =** 22

22= 2× 11

So prime factorization of 198 = 2$\times 9 \times 11 =$ **2** $\times 3^2 \times 11$

9.4 DIVISIBILITY RULE

In Vedic Mathematics the rule of Divisibility helps quickly determine whether a given number is divisible by another number, often using mental calculations. But some times Vedic sutras are used for quicker and more intuitive results.

Divisor	**Vedic Rule**
2	If the last digit is even (0, 2, 4, 6, 8), it's divisible.
3	Add all digits. If the sum is divisible by 3, the number is divisible by 3.
4	Check the last **two digits**. If they form a number divisible by 4, the whole number is divisible
5	If the last digit is 0 or 5, it's divisible.
6	If divisible by both 2 and 3.

7	Twice the last digit then subtracts from the remaining digit Repeat if necessary. If the result is divisible by 7, so the number id divisible.
8	Check the last **three digits**. If they form a number divisible by 8, then it is divisible.
9	Add all digits. If the sum is divisible by 9, the number is divisible by 9.
10	Last digit must be 0.
11	Alternate sum of digits (subtract and add alternately). If the result is divisible by 11, the number is.
12	If divisible by both 3 and 4.

Table 8.4 Divisibility rule in Vedic mathematics

Example: Using Vedic rule of seven check if 672 is divisible by 7.

Solution:

Step 1: Double the last digit → 2 × 2 = 4

Step 2: Subtract from remaining digits: 67 - 4 = 63

Step 3: 63 ÷ 7 = 9 => Divisible

9.5 MODULAR ARITHMETIC

Modular arithmetic in Vedic Mathematics refers to a system of arithmetic for integers, where numbers "round up " when they reach a certain value-known as the modulus.

Checking for Divisibility by 7 and 13 using Modular Arithmetic

Divisibility by 7:
Let N be a number divisible by 7. This means that $N = 7K$ where K is an integer.

We can write

$$N = 10^n a_n + 10^{n-1} a_{n-1} + \cdots + 10a_1 + a_0 = 7K$$
$$N = 10(10^{n-1} a_n + 10^{n-2} a_{n-1} + \cdots + a_1) + a_0 = 7K$$

Let $(10^{n-1} a_n + 10^{n-2} a_{n-1} + \cdots + a_1) = A$ and $a_0 = B$, then $N = 10A + B = 7K$
Adding and then also subtracting $20B$ yields
$N = 10A + B - 20B + 20B = 7K$
which yields
$N = 10(A - 2B) = 7(K - 3B)$
Since, $10\text{mod}7 = 3$, for N to be divisible by 7 , it follows that when the factor $(A - 2B)$ is divided by 7 , the remainder must be 0 , i.e. $(A - 2B) \equiv 0(\text{mod}7)$.

We thus see that if N is divisible by 7 , then $N = (10A + B) \equiv (A - 2B)(\text{mod}7)$.

Example1: Divisibility Test for the Number 8801
The method described below involves attempting to write the number being investigated as a sum of multiples of the divisor (as far as it is possible). The modulo addition property is then employed.

Is 8801 a prime number?
a) Check for divisibility by 7

Attempt to write 8801 as the sum of multiples of 7:
$8801 = 8400 + 350 + 49 + 2$
$8400 \equiv 0(\text{Mod}7), 350 \equiv 0(\text{Mod}7), 49 \equiv 0(\text{Mod}7)$, and $2 \equiv 2(\text{Mod}7)$
By the modulo addition property, $8801 \equiv 2(\text{Mod}7)$, therefore, 8801 is not divisible by 7 .
b) Check for divisibility by 11

Attempt to write 8801 as the sum of multiples of 11:
$8801 = 8800 + 1$
$8800 \equiv 0(\text{ Mod}11\text{ }), 1 \equiv 1(\text{ Mod}11\text{ })$
By the modulo addition property, $8801 \equiv 1(\text{Mod}11)$, therefore, 8801 is not divisible by 11.
c) Check for divisibility by 13

Attempt to write 8801 as the sum of multiples of 13:
$8801 = 7800 + 910 + 91$
$7800 \equiv 0(\text{Mod}13), 910 \equiv 0(\text{Mod}13), 91 \equiv 0(\text{Mod}13)$
By the modulo addition property, $8801 \equiv 0(\text{Mod}13)$, therefore, 8801 is divisible by 13.
Conclusion: 8801 is not a prime number.

Example: Is 2023 divisible by 7 ?
$A = 202, B = 3, (A - 2B) = (202 - 2 * 3) = 196$
The process is now repeated for 196:
$A = 19, B = 6, (A - 2B) = (19 - 2 * 6) = 7$

Thus $(A-2B) \equiv 0(\bmod 7)$, and thus 2023 is shown to be divisible by 7 .
Divisibility by 13 :
Let N be a number divisible by 13 . This means that $N=13K$ where K is an integer.
We can write

$$N=10^n a_n+10^{n-1} a_{n-1}+\cdots+10 a_1+a_0=13K$$
$$\mathrm{N}=10\left(10^{\mathrm{n}-1} \mathrm{a}_{\mathrm{n}}+10^{\mathrm{n}-2} \mathrm{a}_{\mathrm{n}-1}+\cdots+\mathrm{a}_1\right)+\mathrm{a}_0=13\mathrm{K}$$

Let $\left(10^{\mathrm{n}-1} \mathrm{a}_{\mathrm{n}}+10^{\mathrm{n}-2} \mathrm{a}_{\mathrm{n}-1}+\cdots+\mathrm{a}_1\right)=\mathrm{A}$ and $\mathrm{a}_0=\mathrm{B}$, then
$\mathrm{N}=10\mathrm{A}+\mathrm{B}=13\mathrm{K}$
Adding and then also subtracting 40B yields
$\mathrm{N}=10\mathrm{A}+\mathrm{B}-40\mathrm{B}+40\mathrm{B}=13\mathrm{K}$
which yields
$\mathrm{N}=10(\mathrm{A}+4\mathrm{B})=13(\mathrm{K}-3\mathrm{B})$
Since, $10 \bmod 13=10$, for N to be divisible by 13 , it follows that when $(\mathrm{A}+4\mathrm{B})$ is divided by 13 , the remainder must be 0 , i.e. $(A+4B) \equiv 0(\bmod 13)$.

We thus see that if N is divisible by 13 , then $N=(10A+B) \equiv(A+4B)(\bmod 7)$.

Example: Is 50661 divisible by 13?
$A=5066, B=1,(A+4B)=(5066+4^* 1)=5070$
The process is now repeated for 5070: $A=507, B=0,(A+4B)=507$
The process is repeated for 507:

$$A=50, B=7,(A+4B)=78$$

The process is repeated for 78:

$$A=7, B=8,(A+4B)=39$$

If the process is repeated for 39:

$$A=3, B=9,(A+4B)=39$$

we see that 39 is again obtained. The process can now be terminated, since $39 \equiv 0(\bmod 13)$, thus $(A+4B) \equiv 0(\bmod 13)$. Thus 50661 is shown to be divisible by 13. In the test for divisibility of a number $N=(10A+B)$ by 7, the value of $(A-2B)$ is determined. This value is, itself, then written in the form ($10A+B$ ') and the value of ($A'-2B'$) is determined. The process is repeated until a final ($A''-2B''$) value is found for which $(A''-2B'') \equiv 0(\bmod 7)$. In the test for divisibility of a number $N=(10A+B)$ by 13, the value of $(A+4B)$ is determined. This value is, itself, then written in the form ($10A'+B'$) and the value of ($A'+4B'$) is determined. The process is repeated until a final ($A''+2B''$) value is found for which $(A"+4B") \equiv 0(\bmod 13)$. These modulo proofs for divisibility by the primes 7 and 13 can be extended to other prime numbers as well.

Table 8.5.1summarises the ($A \pm nB$) values which are employed in the identification of the twelve (previously unidentified) non-primes in Table 2. The prime divisors have been found to be 7,11,13,17,19,23 and 29.

	Divisor	$(\boldsymbol{A \pm nB})$ value	Initial A	Initial B	Final A	Final B	Final Step
1001	7	$(A - 2B) \equiv 0(\text{mod}7)$	100	1	9	8	$9 - 2 * 8 = -7$
1003	17	$(\text{A} - 5\text{B}) \equiv 0(\text{mod}17)$	100	3	8	5	$8 - 5^* 5 = -17$
1007	19	$(A + 2B) \equiv 0(\text{mod}19)$	100	7	11	4	$11 + 2 * 4 = 19$
1027	13	$(A + 4B) \equiv 0(\text{mod}13)$	102	7	13	0	$13 + 4 * 0 = 13$
1037	17	$(A - 5B) \equiv 0(\text{mod}17)$	103	7	6	8	$6 - 5 * 8 = -34 - 34 \equiv 0(\text{mod}17)$
1043	7	$(A - 2B) \equiv 0(\text{mod}7)$	104	3	9	8	$9 - 2 * 8 = -7$
1057	7	$(A - 2B) \equiv 0(\text{mod}7)$	105	7	9	1	$9 - 2 * 1 = 7 \equiv 0$
1067	11	$(\text{A} - \text{B}) \equiv 0(\text{mod}11)$	106	7	9	9	$9 - 9 = 0$
1073	29	$(A + 3B) \equiv 0(\text{mod}29)$	107	3	11	6	$11 + 3 * 6 = 29$
1079	13	$(\text{A} + 4\text{B}) \equiv 0(\text{mod}13)$	107	9	14	3	$14 + 4 * 3 = 2626 \equiv 0(\text{mod}13)$
1081	23	$(A + 7B) \equiv 0(\text{mod}23)$	108	1	11	5	$11 + 7 * 5 = 4646 \equiv 0(\text{mod}23)$
1099	7	$(A - 2B) \equiv 0(\text{mod}7)$	109	9	9	9	$9 - 2 * 1 = 7$

Table 8.5.1: The application of $(A \pm nB)$ values for prime divisors 7 to 29

It can be observed that every prime divisor has a particular the $(A \pm nB)$ value associated with it.

9.6DIOPHANTINE EQUATION.

Although ancient **Vedic Mathematics** doesn't mention Diophantine equations by name, its **techniques and sutras** provide rapid, **mental methods** for solving such equations—especially **linear Diophantine equations**. The Vedic approach emphasizes, Systematic trials, Working backwards and Elimination methods.

Examples: 1 Sole the linear Diophantine equation 7x + 5y =1.

Solution: Step 1: FirstFind G.C.D of 7 and 5 .

- ⇨ GCD(7,5) =1
- ⇨ Solution Exist if R.H.S is divisible by G.C.D.

Step 2: Express 1 as a linear combination of 7 and 5

We find 1=3× $\mathbf{5 - 2 \times 7}$

So, 1 = 3 (5) - 2 (7)

- ⇨ X=-2 and y=3

Step 3: General solution is

X= -2 + 5 t, y=3-7 t for any integer t.

9.7MULTIPLICATION(URDHVA- TIRAYAK SUTRA).

"Urdhva-Tiryak" means **"Vertically and Crosswise"**.
This is a **general multiplication method** of Vedic Mathematics that works in all situations and helps to do multiplication mentally or in fewer written steps.
Example1: Multiply 23$\times$ $\mathbf{14}$ (2-digit $\times$ $\boldsymbol{2\ digit}$)

Solution: Break the number

2 3 i.e., 20 + 3

× 1 4 i.e., 10 +4

Now we will follow "**Urdhva-Tiryak" steps**

Step 1:vertical →unit digit

3 × **4** =12 → write 2, carry 1

Step 2: Crosswise

(**2**× **4**) + (**3**× **1**) = 8 + 3 =12

Add carry: 11+1 = 12→ write 2, carry 1

Step 3: Vertical →ten digits

2× **1** = 2, plus carry =3

Final answer: 322

9.8 MOUNTAIN EXPANSION

Mountain Expansion is not a formal Vedic sutra (formula), but rather avisual or structural technique sometimes used in explanation of Vedic Math explanations to expand algebraic expressions or numbers. It is used to expand a function especiallybinomials**,** polynomials, or power**s** of numbers near a base like 10, 100, etc.

It derived its name from the triangular (mountain-like**)** structure of the expanded terms, which often resembling Pascal's Triangle**.**

CHECK YOUR PROGRESS

MULTIPLE CHOICE QUESTIONS

Q1.Which of the following is a common Vedic technique used to find prime factors quickly?

A) EkadhikenaPurvena

B) Urdhva-Tiryak Sutra

C) Divisibility rules and mental subtraction

D) Anurupyena

Q2.In Vedic Math, divisibility by 9 can be quickly checked by:

A) Multiplying by 9

B) Adding all digits and checking if the sum is divisible by 9

C) Subtracting last digit from rest

D) Using EkanyunenaPurvena

Q3.In modular arithmetic, what is the remainder when 23 is divided by 7?

A) 3

B) 2

C) 1
D) 5

Q4.Diophantine equations have solutions only when:
A) All terms are divisible
B) The solution is positive
C) The coefficients and constants share common factors
D) The solution is an integer

Q5.The Urdhva-Tiryak Sutra is used for:
A) Division
B) Square roots
C) Vertical and crosswise multiplication
D) Cube roots

Q6.Mountain Expansion in Vedic Math refers to:
A) Expanding logarithms
B) A way to simplify complex equations
C) A visual pattern for binomial expansion
D) Vedic geometry formula

9.9 SUMMARY

This unit shows how **Vedic Mathematics blends ancient knowledge with modern efficiency** to make arithmetic faster, more logical, and easier to understand. Learners explore alternative methods that simplify calculations such as equations, decimals, and large numbers. The unit introduces **Vedic approaches to prime factorization**, where **divisibility rules and mental shortcuts** are used to quickly identify small prime factors like 2, 3, 5, 7, and 11 without lengthy division. Understanding **divisibility rules** helps learners check whether a number is divisible by another number quickly and accurately. This unit are also introduced to **modular arithmetic**, which focuses on remainders and is often called **clock arithmetic**, helping develop a deeper understanding of number cycles and patterns. Finally, the unit covers **multiplication using the Urdhva–Tiryak Sutra**, meaning **"vertically and crosswise."** This powerful technique enables fast multiplication of any numbers while improving mental calculation skills and confidence. Overall, this unit strengthens **number sense, logical thinking, and problem-solving skills**, making mathematics more engaging and efficient for learners.

9.10 GLOSSARY SUMMARY

i. **Anurupyena:** A Vedic Sutra meaning *"proportionately"*, used in simplification and ratios.
ii. **Cross-Multiplication:** A method used in solving equations and partial fractions quickly.
iii. **Diophantine Equation:**An algebraic equation where only **integer solutions** are required
iv. **Divisibility Rule**: Rules that determine whether a number is divisible by another number.

v. **EkadhikenaPurvena:** Sutra meaning *"By one more than the previous"*, useful in recurring decimals and division.

vi. **Modular Arithmetic:** Arithmetic dealing with **remainders** after division.

vii. **Mountain Expansion:**Visual pattern method for **binomial expansions** using symmetric coefficients.

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9.12 SUGGESTED READING

1. **"Vedic Mathematics" by Bharati Krishna Tirthaji :** The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras.Covers mental math techniques for arithmetic, algebra, and geometry

2. "Vedic Mathematics Made Easy" by Dhaval Bathia: A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.

3. **"The Power of Vedic Maths" by Atul Gupta :** Focuses on speed and accuracy.Ideal for practical application in exams like CAT, GRE, and banking tests.

4. **"Mathematics in India" by Kim Plofker** : A scholarly account of the historical development of mathematics in India.Provides context on how Vedic methods evolved over time.

5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** covers Vedic and post-Vedic developments with examples and explanations.

9.13 TERMINAL QUESTIONS

1. Use Urdhva-Tiryak Sutra to multiply 43 × 21……………………………………..

…………………………………………………………………………………………….

2. What is Mountain Expansion? Write the binomial expansion of (a+b)3 using it.

…………………………………………………………………………………………..

3. Use Vedic method to find the prime factors of 84.

…………………………………………………………………………………………….

4. Check whether 4311 is divisible by 9 using Vedic divisibility rules.

………………………………………………………………………………………………

5. What does the Urdhva-Tiryak Sutra mean?

…………………………………………………………………………………………………

6. What is the main condition for a Diophantine equation to have a solution?

………………………………………………………………………………………………….

7. Which Vedic sutra is used to solve simultaneous equations by addition and subtraction?

……………………………………………………………………………………………………

9.14 ANSWERS

MCQ1.C
MCQ2. B
MCQ3.D
MCQ4.D
MCQ5.C
MCQ6.C
MCQ7. C
MCQ8. B

UNIT 10: ESSENTIAL TOOLS OF HIGHER ARITHMETIC

CONTENTS:

10.1 INTRODUCTION

Higher arithmetic in Vedic mathematics is a powerful system that blends ancient wisdom with modern numerical understanding. The Vedic approach simplifies even the most complex arithmetic operations through elegant mental techniques derived from the sixteen *Sutras* and thirteen *Sub-sutras*. These methods focus on speed, accuracy, and intuitive thinking, enabling learners to handle numbers effortlessly and develop sharp analytical skills. This unit, **Essential Tools of Higher Arithmetic**, introduces the fundamental Vedic techniques that form the backbone of all higher-level numerical operations. It begins with basic tools such as place value, number patterns, factorization, and divisibility rules, and gradually connects these ideas with Vedic principles like *Nikhilam*, *Ekādhikena Pūrvena*, *Anurūpyena*, and *Parāvartya Yojayet*. Through these sutras, learners discover how large multiplications, divisions, squares, roots, and recurring decimal conversions can be performed mentally with speed and clarity. The unit emphasizes not only computational skills but also the deeper mathematical insight that Vedic mathematics offers. By understanding the structure of numbers, recognizing hidden patterns, and applying simplified algorithms, learners gain the essential tools required for advanced arithmetic and competitive examinations. These tools also enhance logical reasoning, memory power, and numerical creativity. In this unit we explained about Essential tools of Higher Arithmetic's like Power, Least Common Multiple, Recurring Decimal, Solution of equation and Partial fraction.

10.2 OBJECTIVES

After studying this unit learners will able

1. To expand the foundational skills gained in Vedic Arithmetic – I through advanced techniques.
2. To understand and use divisibility rules and digital root methods for verification and simplification of calculations.
3. To develop speed, accuracy, and mental agility in solving a wide range of arithmetic problems.

10.3 POWER

In mathematics, a **power** means raising a number to an exponent:

a^n= a× a × a…. (n times)

In Vedic Mathematics, there are special tricks and sutras to mentally calculate squares, cubes, and higher powers efficiently — especially for numbers near 10, 100, etc. Here we are giving some vedic sutra for powers:

(i) **Urdhva-Tiryak Sutra** – For fast multiplication (used in squaring and cubing)
(ii) **NikhilamNavatashcaramamDashatah** – Numbers near base (10, 100…)
(iii) **Anurupyena** – Proportionately
(iv) **EkadhikenaPurvena** – "By one more than the previous" (especially for squaring numbers ending in 5)

Square of Numbers Ending in 5

Let say **75^2**.

Sutra:*EkadhikenaPurvena* — Multiply the **first digit** by **one more**, and append **25**

7 × 8 = 56 → append 25 → **5625**

So, 75^2 = 5625

Square of Numbers Near Base (Nikhilam Method)

Example: 103^2 (near base 100)

Base = 100 → Deviation = +3

Step 1: Add deviation → 103 + 3 = 106

Step 2: Multiply base part (106 × 100) = 10600

Step 3: Add square of deviation → $3^2 = 9$

Final Answer: 10600 + 9 = **10609**

So, 103^2 = **10609**

Cube Using Identity

Example: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Use Mountain Expansion or Pascal's Triangle for coefficients

Example: $(10 + 2)^3$
= 1000 + 3×100×2 + 3×10×4 + 8
= 1000 + 600 + 120 + 8 = **1728**
So, 12^3 = **1728**
Square of Numbers Close to 1000 or 100
Use Nikhilam + base shifting logic.

Example: 998^2
Base = 1000 → Deviation = -2
998 − 2 = 996
Square of 2 = 4 → write as 004 (3 digits, since base = 1000)
Final Answer: 996004
998^2 = 996004

Power Type	**Vedic Trick**	**Example**
Square ending in 5	EkadhikenaPurvena	65^2
Square near 100	Nikhilam Sutra	98^2
Cube	Algebraic identity $(a + b)^3$	12^3
Any multiplication	Urdhva-Tiryak	General use

10.4 LEAST COMMON MULTIPLE

First, we will define the what is least common multiple (LCM). TheLeast Common Multiple (LCM) of two or more numbers is the smallest number that is exactly divisible by each of the given numbers.

Vedic mathematics does not provide any specific or unique method for finding the least common multiple (LCM), as seen in traditional methods like prime factorization or division. Instead, it focuses on efficient arithmetic techniques—particularly for multiplication and division—that can be used to streamline and simplify the process of calculating the LCM.

In another way we can say that in Vedic Mathematics, the process of finding the Least Common Multiple (LCM) combines traditional arithmetic rules with simplified mental calculations. Although the concept of LCM is the same as in traditional mathematics, Vedic techniques help in finding it faster through:

The principles of Vedic mathematics can be applied to LCM problems as follows:

(i)Understanding the Concept:

Vedic Mathematics focuses on mental calculations and simplifying steps, which can be helpful when dealing with LCM. The core idea of LCM (finding the smallest number divisible by all given numbers) remains the same. Vedic Mathematics Sutras (formulae) can be used to simplify calculations involved in finding the LCM, particularly when dealing with larger numbers or algebraic expressions.

(ii) Vedic Techniques for LCM:

(a) Simplifying Calculations:

Vedic Maths offers techniques like "By completion or non-completion" (finding number pairs that add up to multiples of 10) or using the "By the deficiency" rule (working with numbers close to a multiple of 10) to simplify addition and subtraction, which are crucial steps in finding the LCM.

(a) Multiplication and Division:

Efficient Vedic techniques for multiplication and division (like those found in the "Vedic Maths Family Lcm Nd HcfWs 2" worksheet) can significantly speed up the process of finding multiples and factors, which are essential for determining the LCM.

(c) Algebraic Applications:

Vedic Mathematics can also be applied to find the LCM of algebraic expressions, using techniques like "By Alternate elimination and retention" (LOPNA-STHAPANABHYAM SUTRA).

Vedic Methods to Find LCM:

Method 1: Prime Factorization Method (Using Vedic Divisibility Rules)

1. **Prime factorize** each number using Vedic techniques.
2. Take the **highest powers of all prime factors** involved.
3. Multiply them to get the LCM.

Example:1 Find LCM of **12** and **18**

Solution: First Prime factors:

$12 = 2^2 \times 3$ and $18 = 2 \times 3^2$

$\rightarrow$ LCM = $2^2 \times 3^2 =$ **36**

Method 2: LCM using Vedic Sutras→While no single sutra is dedicated only to LCM, these are useful:

(i) **"Ekadhikena Purvena"** – helpful in simplifying factors

(ii) **"Nikhilam Navatashcaramam Dashatah"** – used in multiplication/division to speed up calculations

(iii) **"Urdhva Tiryak Sutra"** – used in crosswise multiplication when checking common multiples

Example 2: Find the LCM of 20, 30, and 50 using Vedic prime factorization.

Solution: First we will find prime factor

$20 = 2^2 \times 5$

$30 = 2 \times 3 \times 5$

$50 = 2 \times 5^2$

$\rightarrow$ LCM = $2^2 \times 3 \times 5^2 = 300$

10.5 RECURRING DECIMAL.

A recurring decimal is a decimal number in which a digit or group of Digits repeats infinitely.

Examples:

0.333... (recurring digit: 3)

0.142857142857... (recurring group: 142857)

In Vedic mathematics, recurring (or repeating) decimals are handled using intelligent and simplified mental techniques, allowing quick recognition, conversions and operations with such numbers.

Though Vedic mathematics does not prescribe any specific formula for recurring decimals, it does provide quick techniques based on algebraic patterns and fractions.

10.6 SOLUTION OF EQUATIONS

Vedic mathematics provides quick and elegant techniques for solving linear, quadratic and simultaneous equations using mental arithmetic and formulas (sutras). These methods reduce lengthy algebraic steps and emphasize pattern recognition, cross-multiplication and balancing. Here we are giving some type of equation:

(i) LINEAR EQUATIONS

Example:1 Solve 3x+4=103x + 4 = 103x+4=10

Solution: Vedic Method (Transpose and Adjust)
→ Transpose 4 to the right:
$3x = 10 - 4 = 6$
→ $x = 6 \div 3 = 2$

(ii) Simultaneous Linear equation: Using **Sankalana – Vyavakalanabhyam** Sutra

Example1: Solve $\begin{cases} \mathbf{2x + 3y = 17} \\ \mathbf{3x + 2y = 16} \end{cases}$

Solution:
Step 1: Add the both equation
$2x + 3y + 3x + 2y = 17 + 16$
$\Rightarrow 5x + 5y = 33$
Step 2: Subtract the both equation

$$2x + 3y - 3x - 2y = 17 - 16$$

=> -x + y = 1

Step 3: Substitute in first equation

2 x + 3 (x+1) =17

→ 2 x +3 x + 3 =17

=> 5 x = 14

x = 14/5

-y = x+1= $\frac{14}{5} + 1$ =$\frac{19}{5}$

(iii) QUADRATIC EQUATIONS: Use **"Paravartya Yojayet"** for transposition and **"Anurupyena"** for proportion.

Example 1: Solve x^2 -5 x +6 = 0

Solution:x^2 -5 x +6 = 0

First Factorized directly using Vedic pattern:

Find two numbers whose product is 6 and sum is 5 → 2 and 3

→ (x−2) (x−3) = 0

→ -x = 2, 3

If it is not factorable easily, use completing the square or Vedic quadratic formula, applying mental math tricks.

(iv) Equation with fraction: Using Urdhva-Tiryak (vertically and crosswise)

Example: Solve $\frac{x+3}{2} = \frac{x-2}{3}$

Cross-multiply using Urdhva-Tiryak (vertically and crosswise)

→3(x+3)=2(x−2)

→ 3 x + 9 = 2 x – 4 ⇒ x = −13

(v) SPECIAL EQUATIONS (Zero Principle)

Using **"Sunyam Samyasamuccaye"** Sutra: *If the total on both sides is the same, one factor is zero.*

Example:

(x +2)(x−3)=(x+2)(x+1)

Cancel the common factor

(x + 2) → x−3= x + 1⇒ − 3 = 1 (Contradiction → no solution)

Then, **Sunyam Samyasamuccaye**⇒ LHS = RHS, ⇒ All values satisfy equation (infinite solutions).

10.7 PARTIAL FRACTION

If a fraction is in the form of a polynomial and can be broken down into smaller simpler fractions, then it is called a partial fraction.

$$\frac{2x + 3}{(x+1)(x+2)}$$

Some important formulas of Vedic mathematics are used in this method:

Paravartya Yojayet: "Add by changing place" - We use this to remove or make the denominator zero.

Anurupyena:"In proportion" - means dividing in a proportionate manner.

Sunyam Samyasamuccaye: If numerator and denominator have the same set then it becomes zero.

Example: Decompose

$$\frac{2x + 3}{(x+1)(x+2)}$$

Solution: Let us assume

$$\frac{2x+3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

For solving the problem with the help of Vedic mathematics

First, we will take denominator (x+1) (x+2)

Cover-up method (similar to modern Heaviside technique but rooted in Vedic “Paravartya”):

For A: cover x+1, put x =-1 → A=1

For B cover x+2, put x = -2→ B =1

We can divide it into smaller fractions.

CHECK YOUR PROGRESS

MULTIPLE CHOICE QUESTIONS

1. **In Vedic math, partial fractions are simplified using:**

A)Urdhva-Tiryak Sutra
B) Cross-multiplication and smart substitution
C) Factor theorem
D) Completing the square

2. If the product of LCM and HCF of a number is equal to a×b, then what is this rule based on?

A) Division rule
B) Euclid's rule
C) Fundamental theorem of arithmetic
D) Prime factorization

10.8 SUMMARY

In this unit, learners study important number concepts that help simplify complex calculations. The concept of **powers (exponents)** is introduced to show how a number can be multiplied by itself repeatedly, making it easier to work with large values. Learners also understand **LCM (Least Common Multiple)**, which is the smallest number that is divisible by two or more given numbers. This concept is useful in solving problems involving fractions, ratios, and real-life situations. The unit explains **recurring decimals**, where a digit or group of digits repeats continuously in a decimal number, helping learners connect fractions and decimals more clearly. Finally, learners are introduced to **partial fractions**, which involve breaking a complicated fraction into simpler parts. Using the **Parāvartya Yojayet (Cover-Up Method)** from Vedic Mathematics, learners can solve partial fractions quickly and efficiently.

10.9 GLOSSARY

i. **Paravartya Yojayet**: Sutra meaning *"Transpose and adjust"*, used in solving equations.
ii. **Recurring Decimal:**A decimal number with digits that **repeat in a cycle**.
iii. **Urdhva-Tiryak Sutra:** Means *"Vertically and crosswise"*. A general multiplication shortcut.
iv. **Vinculum:** A bar used in Vedic Math to **denote negative digits** for simplification.

10.11 REFERENCES

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10.12 SUGGESTED READING

1. **"Vedic Mathematics" by Bharati Krishna Tirthaji:** The foundational text on Vedic Mathematics, written by the discoverer of the 16 sutras.Covers mental math techniques for arithmetic, algebra, and geometry
2. "Vedic Mathematics Made Easy" by Dhaval Bathia: A simplified, beginner-friendly version of Vedic techniques.Great for learners and competitive exam aspirants.
3. **"The Power of Vedic Maths" by Atul Gupta:** Focuses on speed and accuracy.Ideal for practical application in exams like CAT, GRE, and banking tests.
4. **"Mathematics in India" by Kim Plofker**: A scholarly account of the historical development of mathematics in India. Provides context on how Vedic methods evolved over time.
5. **"Ancient Indian Leaps into Mathematics" by B. S. Yadav and Manju Bhargava:** Covers Vedic and post-Vedic developments with examples and explanations.

10.13 TERMINAL QUESTIONS

1. Find the LCM of 15, 20 and 25.

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2. If HCF of two numbers = 6 and LCM = 72, then what will be the product of those numbers?

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……………………………

3. A train whistles every 12 minutes and another train whistles every 18 minutes. Both blow their whistles together at 6 o'clock. At what time will both trains whistle together next time?

……………………………………………………………………………………………………

10.14 ANSWERS

MCQ1.B
MCQ2. D

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