A–0720

Total Pages : 4 **Roll No. ....**

MT-606

MA/MSC Mathematics (MAMT/MSCMT)

(Analysis and Advanced Calculus-II)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note :– This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note :– Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that If H be a given Hilbert space and T* be adjoint of the operator T. Then T* is a bounded linear transformation and T determines T* uniquely.

2. Prove that If T is a normal operator on Hilbert space H then each eigenspace of T reduces T.

3. Prove that Let f be C^1 map on a compact interval $[a, b]$ into a compact interval $[c, d]$ of R and let g be a continuous function on $[c, d]$ into a Banach space X over K. Then :

$$\int_a^b (\mathrm{D}(f(s)))\, g(f(s))ds = \int_{f(a)}^{f(b)} g(t)cdt$$

4. State and prove Implicit function Theorem.

5. If X be a Banach space over the field K of scalars and let :

$$f : [a, b] \to \mathrm{X},\ g : [\mathrm{a}, \mathrm{b}] \to \mathrm{R}$$

be continuous and differentiable functions such that $\| \mathrm{D}f(t) \| \leq \mathrm{D}g(t)$ at each point $t \in (a, b)$. Then prove that $\| f(b) - f(a) \| \leq g(b) - g(a)$.

Section–B

(Short Answer Type Questions) $4 \times 8 = 32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define Adjoint Operator, Self-Adjoint Operator and Normal operator.

2. Define derivative of a map in Banach spaces and show that derivative of the constant function is the zero linear map and derivative of a continuous linear mapping is the mapping itself.

3. Prove that Let f be a regulated function on a compact interval $[a, b]$ of R into a Banach space X then at each $t \in [a, b]$, the t function $F : [a, b] \to X$, $F(t) = \int_0^t f$, $t \in [a, b]$ is continuous.

4. Prove that if T is an operator on a Hilbert space H, then T is normal iff its real and imaginary parts commute.

5. Prove that if T_1 and T_2 are normal operators on H with the property that either commute with adjoint of the other, than $T_1 + T_2$ and $T_1 T_2$ are normal.

6. State and proof spectral theorem.

7. Prove that If T is a normal operator on a Hilbert space H, then x is an eigenvector of T with eigen value α, iff x is an eigenvector of T*. with $\overline{\alpha}$ as an eigenvalue.

8. Proof that if x is an eigenvector of T, then x cannot correspond more than one eigenvalue of T.

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