Total Pages : 3 **Roll No. ....**

MT-501

MA/MSC Mathematics (MAMT/MSCMT)

(Advanced Algebra-I)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (i) If $f : G \to G'$ be a homomorphism of groups and $e_1 e^1$ be the identities in G and G^1 respectively. Then :

   (a) $f(e) = e^1$

   (b) $f(a^{-1}) = [f(a)]^{-1}$ where $a \in G$.

   (ii) A homomorphism f of a group G into a group G^1 is an isomorphism if and only if Ker $f = \{e\}$.

2. Define direct sum of a vector subspace. If $V = R^3$ and w_1 is the xy plane and w_2 is yz plane then prove that v is not the direct sum of w_1 and w_2.

3. Define Range of homomorphism. Prove that range of homomorphism of module is also submodule ?

4. Under what conditions on the scalar 'a' are the vectors $(a, 1, 0)$, $(1, a, 1)$ and $(0, 1, a)$ in R^3 linearly dependent.

5. Define algebraic extension of a field. Prove that every finite extension of a field is an algebraic extension.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that every non-zero element in a field is a unit.

2. Under what conditions of a scalar 'm^1' are the vectors $(m, 1, 0)$ $(1, m, 1)$ and $(0, 1, m)$ in R^3, linearly independent ?

3. Define solvable group. Prove that symmetic group of degree 3 is solvable.

4. Prove that every field is an Euclidean ring.

5. Define the following :

   (i) Conjugate class

   (ii) Self conjugate

6. Check the function T from R^2 into R^2 is linear transformation or not :

$$T(x_1, x_2) = (\sin x_1, x_2)$$

7. Show that the group of integers z has no composition series ?

8. Prove that any product of solvable group is solvable ?

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