A–0690

Total Pages : 4 **Roll No. ....**

MAT-602

M.Sc. Math (MSCMT)

(Functional Analysis)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that addition is a continuous function in a normed linear space.

2. State and proof F. Riesz's Lemma.

3. Let T be a linear operator. Then prove that :

   (a) The range R(T) is a vector space.

   (b) If dim D(T) = $n < \infty$, then dim R(T) $\leqq n$.

   (c) The null space N(T) is a vector space.

4. Let X be an inner product space and let $x, y \in$ X. Then, prove that :

$$\| x + y \|^2 + \| x - y \|^2 = 2(\| x \|^2 + \| y \|^2)$$

5. Let (X, $\| \;\; \|$) be a normed linear space over a field K (=R or C). Let x_1 and x_2 be two vectors in X such that $x_1 \neq x_2$. Then there is a bounded linear functional F on X such that F(x_1) $\neq$ F(x_2).

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let p be a real number such that $1 \leq p < \infty$. Show that the space l_p^n of all n–tuples of scalars with the norm defined by :

$$\| x \|_p = \{\Sigma_{i=1}^{n} | X_i |^p\}^{\frac{1}{p}}$$

is a Banach space.

2. A compact subset M of a metric space is closed and bounded.

3. Show that the mapping $T : V_2(R) \rightarrow V_3(R)$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(R)$ into $V_3(R)$.

4. Prove that every contraction mapping is continuous.

5. Explain the statement of Baire's Category Theorem and Uniform Boundedness Theorem.

6. Explain the statement of Hahn Banach Theorem for Real Vector Space and Hahn Banach Theorem for Complex Vector Space.

7. Write the Schwarz inequality and triangle inequality.

8. Let $X = \mathbb{C}$ (set of complex numbers) $x = a + ib \in \mathbb{C}$,

   prove that $(\mathbb{C}, \| x \|)$ is a normed linear space.

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