A–0688

Total Pages : 3 **Roll No. ....**

MAT-509

Mathematics (MSCMAT/MAMT)

(Mathematical Methods)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve the following integral equation by successive approximations :

$$y(x) = 1 + \int_0^x y(t)\, dt$$

taking $y_0(x) = 0$.

2. Find the Green's function of the boundary value problem $y'' = 0,\ y(0) = y(1) = 0$.

3. Convert the following differential equation into integral equation :

$$y'' + \lambda y = 0 \text{ when } y(0) = 0,\ y(1) = 0$$

4. State and prove the necessary condition for the existence of extremal.

5. Extremize :

$$\mathrm{I}[y(x)] = \int_0^1 (\sqrt{1 + y'^2})\, dx$$

with $y(0) = 0,\ y(1) = 1$.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. If $\mathrm{F}(t) = \dfrac{e^{at} - \cos \omega t}{t}$, find the Laplace transform of $\mathrm{F}(t)$.

2. Find the inverse Laplace transform of $\dfrac{p^3}{p^4 - a^4}$.

3. Obtain the Fourier series to represent $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

4. Solve the Homogeneous Fredholm equation :

$$y(x) = \lambda \int_0^1 e^x e^t y(t)\, dt$$

5. Show that the function $y(x) = (1 + x^2)^{-3/2}$ is a solution of the Volterra integral equation :

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)\, dt$$

6. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

7. Solve the boundary value problem $y'' - y + x = 0$. $(0 \leq x \leq 1)$ subject to boundary conditions $y(0) = 0$, $y(1) = 0$ by Rayleigh-Ritz method.

8. If $\mathrm{F}(t) = \dfrac{e^{at} - \cos \omega t}{t}$, find the Laplace transform of $\mathrm{F}(t)$.

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