A–0685

Total Pages : 4 **Roll No. ....**

MAT-506

M.Sc. Math (MSCMT)

(Topology)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Show that if $B_1, B_2, \ldots\ldots\ldots\ldots, Bn \in \mathbf{B}$ (Basis) and element $x \in B_1 \cap B_2 \cap \ldots\ldots\ldots \cap B_n$. Then there exists some member $B' \in \mathbf{B}$ (Basis) such that :

$$x \in B' \subseteq B_1 \cap B_2 \cap \ldots\ldots\ldots \cap B_n$$

2. Prove that a topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in X.

3. Show that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

4. Show that Compactness implies limit point compactness, but not conversely.

5. Prove that Every compact Hausdorff topological space $(X, \mathcal{T})$ is a regular space.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Consider, $X \neq \emptyset$. Show the collection $\mathbf{B} = \{\{x\} \mid x \in X\}$ is a basis.

2. Let $\mathcal{T}_1 = \{\emptyset, \{1\}, X_1\}$ be a topology on $X_1 = \{1,2,3\}$ and $\mathcal{T}_2 = \{\emptyset, X_2, \{a\}, \{b\}\ \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ be a topology for $X_2 = \{a, b, c, d\}$. Find a base for the product topology $\mathcal{T}$?

3. Show that let $(X, \mathcal{T})$ be a topological space and $A \subseteq X$, then A is closed iff $A' \subseteq A$, or $A \supseteq D(A)$. A subset A of X in a topological space $(X, \mathcal{T})$ is closed iff A contains each of it's limit points.

4. Prove that let $\rho : X \to Y$ be a surjective function, where X is a topological space. The quotient topology is the finest topology on Y such that $\rho : X \to Y$ is continuous.

5. On a topological space X, define a relation xRy, if there is a connected subset of X containing both x and y. Show that this relation is an equivalence relation.

6. Show that if $A = (0, 1)$ on the real line R with the usual topology then A is not compact.

7. Show that the rationals Q is not locally compact.

8. Give an example to show that subspace of a separable topological space need not be separable.

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