A–0715

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MT-601

MA/MSC Mathematics (MAMT/MSCMT)

(Analysis and Advanced Calculus-I)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If $x, y, a \in X$ and k is any scalar. Prove that a metric d induced by a norm on a normed space N satisfies :

   (i) $d(x + a, y + a) = d(x, y)$

   (ii) $d(kx, ky) = |k| d(x, y)$

2. Let $L_2[0,1]$ be the set of all square integrable functions on [0, 1]. Define the inner product on $L_2[0, 1]$ as :

$$\langle f, g \rangle = \int_0^1 g(t)\overline{f(t)}dt, \ \forall f, g \in L_2[0,1]$$

   Prove that L_2 [0, 1] is an inner product space . Check is this a Hilbert space ?

3. Prove that l_p is a Banach space, $1 \le p < \infty$.

4. If M be a closed proper subspace of a normed linear space N and a is a real number such that

$$0 < a < 1, \text{ then } \exists \text{ a vector } x_0 \in N \text{ s. t.}$$

$$\| x_0 \| = 1 \text{ and } \| x - x_0 \| \ge a \, \forall x \in M$$

5. State and prove Risez Representation theorem.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define a normed linear spaces and show that every convergent sequence is a Cauchy sequence.

2. If x and y are any two vectors in a Hilbert space, then prove that :

(i) $\| x + y \|^2 + \| x - y \|^2 = 2[\| x \|^2 + \| y \|^2]$

(ii) $4\langle x, y\rangle = \| x + y\|^2 - \| x-y \|^2 + i\| x+iy \|^2 - i\| x - iy \|^2$

3. State and prove closed graph theorem.

4. Show that dual of R^2 is R^2.

5. Let X be a complex inner product space, then show that :

(i) $\langle \alpha x - \beta y, z\rangle = \alpha\langle x, z\rangle - \beta\langle y, z\rangle$

(ii) $\langle x, \beta y + \gamma z\rangle = \overline{\beta}\langle x, y\rangle + \overline{\gamma}\langle x, z\rangle$

(iii) $\langle x, \beta y - \gamma z\rangle = \overline{\beta}\langle x, y\rangle - \overline{\gamma}\langle x, z\rangle$

(iv) $\langle x, 0\rangle = 0$ and $\langle 0, x\rangle = 0 \ \forall \ x \in X$

6. Prove that a complete inner product space is a Hilbert space.

7. State and prove Bessel 's inequality.

8. Let y be a fixed elements of Hilbert space H and f_y be a scalar valued functional on H defined as $f_y(x) = \langle x, y \rangle$, $\forall x \in H$. Then show that the mapping f_y is a functional on :

$$\| y \| = \left| f_y \right|$$

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