A–0713

Total Pages : 4 **Roll No. ....**

MT-509

MA/MSC Mathematics (MAMT/MSCMT)

(Differential Geometry and Tensor-II)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Derive the general differential equations of a geodesic on the surface $r = r(u,v)$.

2. State and Prove Gauss's Characteristic equation.

3. (a) A covariant tensor of first order has components xy, $2y - z^2$, xz in rectangular coordinates. Determine its covariant components in spherical polar coordinates.

   (b) Prove that the outer multiplication of tensors is commutative and associative.

4. Show that the metric of a Euclidean space, referred to cylindrical coordinates is given by :

$$ds^2 = (dr)^2 + (r\, d\,\theta)^2 + (r \sin\theta\, d\phi)^2$$

Determine its metric tensor and conjugate metric tensor.

5. State and prove Schur's theorem.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that the curves $u + v$ = constant are geodesics on a surface with metric :

$$(1 + u^2)du^2 - 2uv \text{ and } v + (1 + v^2)dv^2$$

2. Show that for the right helicoid :

$$\vec{r} = (u \cos v, u \sin v, cv)$$

$$l = 0, m = 0, n = -u; \lambda = 0, \mu = \frac{u}{(n^2 + c^2)}, v = 0$$

3. Define the following :

   (a) Kronecker delta

   (b) Contravariant vector

4. Calculate the Christoffel symbols corresponding to the metric :

$$ds^2 = (dx^1)^2 + G(x^1, x^2)(dx^2)^2$$

where G is a function of x^1 and x^2.

5. Evaluate div Aj in (i) cylindrical polar coordinates, and (ii) spherical polar coordinates.

6. If the intrinsic derivative of a vector A^i along a curve C vanishes at every point of the curve, then show that the magnitude of the vector A^i is constant along the curve.

7. Show that the necessary and sufficient conditions that a system of coordinates be geodesic with the pole P_0 are that their second covariant derivatives, with respect to the metric of the space, all vanish at P_0.

8. Prove that the unit tangent vectors form a field of parallel vectors along a geodesic.

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