**A–0712**

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# MT-508

## MA/MSC Mathematics (MAMT/MSCMT)

**(Special Functions)**

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

***Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

### Section–A

**(Long Answer Type Questions)** 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$x.\frac{d^2y}{dx^2}+2.\frac{dy}{dx}+xy=0$$

by Frobenius method.

2. Show that :

(a) $${}_1F_1(a:b;z)=\frac{1}{(a)!}\int_0^\infty e^{-t}t^{a-1}{}_0F_1(-;b;zt)dt$$

(b) $${}_1F_1(a;b;z)=e^z{}_1F_1(b-a;b;-z)$$

3. Show that :

(a) $\sin(z\sin\theta) = 2\sin\theta.J_1 + 2\sin 3\theta\, J_3 + .....$

(b) $\cos(z\sin\theta) = J_0 + 2\cos 2\theta.J_2 + 2\cos 4\theta.J_4 + .....$

4. Evaluate :

$$\int_{-\infty}^{\infty} x^2 e^{-\dot{x}}\{H_n(x)\}^2 dx$$

$$=\sqrt{\pi}.2^n.(n)!.\left(n+\frac{1}{2}\right)$$

5. Show that :

(a) $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$

(b) $L'_n(x) = -\sum_{r=0}^{n-1} L_r(x)$

## Section–B

**(Short Answer Type Questions)** 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Write a short note on the following :

   (a) Linear transformation

   (b) Hypergeometric functions

2. Prove :

$$\int_{-1}^{1} P_m(x)P_n(x) = 0\,, \text{ if } m \neq n$$

3. Define Kummer's confluent hypergeometric function. Write down two properties of it.

4. Show that :

$$n.P_n = (2n - 1)xP_{n-1} - (n - 1)P_{n-2},$$

   where $n \geq 2$.

5. Write down the Gauss hypergeometric differential equation. What are its singular points.

6. Prove that :

$$\lim_{z \to 0} \frac{J_n(z)}{z^n} = \frac{1}{2^n.(n+1)!}, \quad \text{for } n > -1.$$

7. Express :

$$H(x) = x^4 + 2.x^3 + 2.x^2 - x - 3,$$

in terms of Hermite's polynomials.

8. Show that :

$$L_n^n(0) = (-1)^n$$

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