

A-0711

Total Pages : 4

Roll No.

MT-507

MA/MSc Mathematics (MAMT/MScMT)

(Topology)

Examination, June 2025

Time : 2:00 Hrs.

Max. Marks : 70

Note :- This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. *Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.*

Section-A

(Long Answer Type Questions) 2×19=38

Note :- Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

A-0711/MT-507

(1)

P.T.O.

1. Define a topological space and explain the properties of open sets in detail. What are closed sets in a topological space ? Prove that arbitrary intersection of closed sets is closed.
2. Define bases and sub-bases for a topology. Prove that the topology generated by a sub-basis is the smallest topology containing all the sets in the sub-basis.
3. Define a continuous function between two topological spaces. Prove that the preimage of a closed set under a continuous function is closed. Provide an example of a continuous function between two spaces and show why it is continuous ?
4. Define a compact space. Prove that a subset of \mathbb{R}^n is compact if and only if it is closed and bounded.
5. Define the quotient topology on a set, and prove that the quotient map is continuous. Give an example of a quotient space that is not Hausdorff.

Section–B

(Short Answer Type Questions) 4×8=32

Note :– Section ‘B’ contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define compact spaces. Prove that every closed subset of a compact space is compact.
2. Prove that the interior is an open set and the closure is a closed set.
3. Define homeomorphism. Give examples :
 - (i) Two spaces that are homeomorphic to each other.
 - (ii) Two spaces that are not homeomorphic to each other.
4. What is a locally connected space ? Provide examples without expansion :
 - (i) Space which is connected but not locally connected.
 - (ii) Space which is locally connected but not connected.
 - (iii) Space which is both connected and locally connected.
5. Define connectedness. Can the union of two connected spaces be disconnected ? What about intersection ? Explain with examples.

6. Define a Hausdorff (T_2) space and give an example of a Hausdorff space and a non-Hausdorff space.
7. What is a net in a topological space ? How is it different from a sequence ?
8. Show that every metric space is T_2 .
