A–0710

Total Pages : 4 **Roll No. ....**

MT-506

MA/MSC Mathematics (MAMT/MSCMT)

(Advanced Algebra-II)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note :– This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note :– Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (a) Prove that any algebraic extension of a finite field F is separable extension.

   (b) Every field of characteristic zero is perfect.

2. Let K be a Galois extension of a field F. Then there exists a one-to-one correspondence between the set of all subfields of K containing F and the set of all subgroups of G(K|F). Further, if E is any subfield of K containing F, then prove that :

   (i) [K : E] = o[G(K|E)] and [E : F] = index of G(K|E) in G(K|F)

   (ii) E is normal extension of F iff G(K|E) is a normal subgroup of G(K|F)

3. Let V and V′ be n-and m-dimensional vector spaces over a field F. For given bases B of V and B′ of V, prove that the function assigning to each linear transformation $t : V \to V'$ its matrix $M_{B'}^{B}(t)$ relative to bases B, and B′ is an isomorphism between the vector space Hom(V, V′) and the space $F^{m \times n}$ of all $m \times n$ matrices over F.

4. (a) Prove that for any matrix A, the row rank of A equals to the column rank of A.

(b) Prove that an n x n square matrix A over field F is invertible iff rank (A) = n.

5. Let V be an inner product space. For any two vectors v, $u \in$ V. prove that the following :

(a) $\|u+v\|^2 - \|u-v\|^2 = 4 < u, v >$

(b) $\|u+v\|^2 - \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let F be a field and let $f(x)$ be a nonzero polynomial in F[x]. Prove that the Splitting field of $f(x)$ is an algebraic extension of F.

2. Define the following :

(a) Characteristic polynomial

(b) Determinant rank

3. State and prove Cayley Hamilton theorem.

4. Prove that a linear transformation t from a finite-dimensional inner product space V to itself is skew-symmetric iff they commute with its adjoint.

5. Prove that if t_1 and t_2 are linear transformation from a finite-dimensional inner product space V to V′, then $(t_1 o t_2)^* = t_1^* o t_2^*$.

6. Let V and V′ be inner product spaces. Prove that every orthonormal linear transformation $t : V \to V'$ preserves the length and angle between two vectors.

7. Prove that the eigenvalues of a self-adjoint linear transformation are real.

8. Define the following :

   (a) Galois group

   (b) Radical extension

****