A–0706

Total Pages : 3 **Roll No. ....**

MT-502

MA/MSC Mathematics (MAMT/MSCMT)

(Real Analysis)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note :– This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note :– Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that if $\{E_n\}$ be a countable collection of sets of real numbers. Then :

$$m^*(\cup_n E_n) \leq \Sigma_n m^*(E_n)$$

2. Prove that if f and g are two functions defined on the common domain E and f is measurable on E. If $f = g$, a.e. (almost everywhere) on E, then g is also a measurable function on E.

3. If f is a bounded function defined on a measurable set E, and $m(E) = 0$. Then show that :

$$\int_E f(x)dx = 0$$

4. Let X be an infinite set and **B** be the collection of all subsets A of X such that either A or A^c is finite. Show that **B** is an algebra but is not a σ-algebra.

5. Prove that if $\langle f_n \rangle$ is a sequence of nonnegative extended real valued measurable functions. Then :

$$\int \underline{\lim} f_n \leq \underline{\lim} \int f_n$$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that every bounded measurable function f defined on a measurable set E is L – integrable over E.

2. Proof that every open interval is a Borel set.

3. Proof that the intersection of two measurable sets is a measurable set.

4. Prove that the characteristic function ϕ_A of a set A is measurable if and only if A is a measurable set.

5. If a constant function on a measurable set E, where $f(x) = c$ for all $x \in$ E. Then prove that :

$$\int_E f(x)dx = c.m(E)$$

6. Show that, for any set A and any $\in > 0$, there is an open set 0 containing A and such that $m^*(0) \leq m^*(A) + \in$.

7. Suppose f is a measurable function and $f = g$ a.e., (almost everywhere) then prove that g is also a measurable function.

8. Prove that a countable set has Lebesgue outer measure zero.

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