A–0691

Total Pages : 7 **Roll No. ....**

MAT-603

M.Sc. Mathematics (MSCMT)

(Operations Research)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. The necessary and sufficient condition for the existence and non-degeneracy of all possible basic feasible solutions of $Ax = b$, $x \geq 0$ is the linear independent of every set of m columns of the augmented matrix $[A, b]$, where A is the $m \times n$ coefficient matrix.

2. Using two-phase method solve the following LP problem :

   Maximize :

$$Z = 5x_1 - 4x_2 + 3x_3$$

   Subject to :

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

3. In the given LPP evaluate the optimum integer solution.

   Maximize :

$$Z = x_1 + 4x_2$$

   Subject to :

$$2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \in Z$$

4. Obtain the initial basic feasible problem of the transportation problem using Vogel's approximation method :

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} 20 & 25 & 28 & 31 \\ 32 & 28 & 32 & 41 \\ 18 & 35 & 24 & 32 \end{bmatrix} \end{array} \begin{array}{c} \text{Supply} \\ 200 \\ 180 \\ 110 \end{array}$$

5. MCS Inc. is a software business that is working on three Y2K projects with the Maharashtra government's departments of housing, education, and health. The project leaders' performance varies across different projects, depending on their expertise and background. Below is the performance score matrix :

Project Leader	**Projects**		
	Health	**Education**	**Housing**
P_1	20	26	42
P_2	24	32	50
P_3	32	34	44

Determine the optimal assignment that maximizes the total performance score.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. In a particular factory, three machines, namely M_1, M_2, and M_3, are utilized in the manufacturing process of two products, P_1 and P_2. Machine M_1 is occupied for 5 minutes for producing one unit of P_1, while M_2 is used for 3 minutes and M_3 for 4 minutes. For one unit of P_2, the time requirements are 1 minute for M_1, 4 minutes for M_2, and 3 minutes for M_3. The profit earned per unit is ₹ 30 for P_1 and ₹ 20 for P_2, regardless of whether the machines operate at full capacity. How can we determine the production plan that maximizes profit ? Solve the linear programming using graphical method.

2. Using simplex method solve the following LPP :

   Maximize :

$$Z = x_1 - 3x_2 + 2x_3$$

Subject to :

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

3. Define the followings :

   (i) Degenracy in linear programming

   (ii) Two phase method

   (iii) Duality

   (iv) Sensitivity analysis

4. Using method of penalty (or Big M) solve the following LP problem :

   Maximize :

$$Z = 6x_1 + 4x_2$$

   Subject to :

$$2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

   Is the solution unique ? If it is not, then find two different solutions.

5. Using two-phase method solve the following LP problem :

Maximize :

$$Z = 5x_1 + 3x_2$$

Subject to :

$$2x_1 + x_2 \leq 1$$

$$3x_1 + 4x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

6. Find the dual of the following linear programming problem :

Maximize :

$$Z = 5x_1 + 3x_2$$

Subject to :

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0$$

7. Examine how the optimal solution is influenced by discrete alterations in the requirement vector for the given Linear Programming Problem (LPP) :

Maximize :

$$Z = 2x + y$$

Subject to :

$$3x + 5x \leq 15$$

$$6x + 2y \leq 24$$

$$x \geq 0, y \geq 0$$

8. Derive the necessary conditions for the non-linear programming problem :

Minimize :

$$Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$$

Subject to the constraints :

$$x_1 + x_2 + x_3 = 11$$

$$x_1, x_2, x_3 \geq 0$$

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