A–0689

Total Pages : 4 **Roll No. ....**

MAT-601

M.Sc. Math (MSCMT)

(Advanced Complex Analysis)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State the argument principle and Rouche's theorem. Find number of roots of the equation $z^5 + 15z + 1 = 0$ inside :

$$\left\{ z \in \mathbb{C} : \frac{3}{2} < |z| < 2 \right\}$$

2. State and prove Carorati-Weierstrass theorem.

3. State Schwarz lemma. Let $D(a, R) := \{z \in \mathbb{C} : |z - a| < R\}$. If $f : D(a, R) \to \mathbb{C}$ is analytic and satisfies $|f(z)| \le M$ and $f(a) = 0$, then use the Schwarz lemma to show that :

$$|f(z)| \le \frac{M|z-a|}{R}$$

4. Show that $u(x, y) = x^3 - 3xy^2$ is a harmonic function. Determine harmonic conjugate $v(x, y)$ of $u(x, y)$, hence find the corresponding holomorphic function $f(z)$ in terms of z.

5. Evaluate :

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+2x+2)}$$

Section–B

(Short Answer Type Questions) $4\times8=32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Mobius transformation which maps i, $-i$, 1 to 0, 1, ∞ respectively.

2. Evaluate :

$$\int_{\gamma} \frac{\sin 3z}{z+\frac{\pi}{2}}\, dz$$

where the trace γ^* of γ is the circle $|z| = 5$.

3. Let $\mathbf{D} := \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $\{a_n = 1/n\}$. Find the holomorphic function $f : \mathrm{D} \to \mathbb{C}$ with the property that the set of zeros of f contains all a_n.

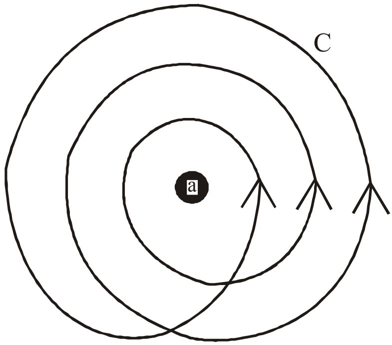
4. Let $f : \mathbb{C} \to \mathbb{C}$ be a holomorphic function such that the real part of f is bounded. What can you say about f ? Explain.

5. Show that a non-polynomial entire function must have an essential singularity at ∞.

6. Suppose f is holomorphic function on a domain D such that $|f| = a$ for some $a \in \mathbb{R}$. Then show that f is constant.

7. Is the function $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = \bar{z}$ holomorphic ? Justify your answer.

8. Evaluate the following integral $\int_C \frac{dz}{z-a}$, where C is the closed curve as shown in the following figure, and 'a' is a point inside C.



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