A–0684

Total Pages : 4 **Roll No. ....**

MAT-505

M.Sc. Math (MSCMT)

(Advanced Linear Algebra)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Consider a vector space M_n (R). Then prove which of the following is/are subspace of M_n (R) :

(i) $W_1 = \{A \in M_n (R) : A = bA\}; b \in R$

(ii) $W_2 = \{A \in M_n (R) : A = A'\}$

(iii) $W_3 = \{A \in M_n (R) : \det (A) = 0\}$

2. Let U and V be vector space over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional. Then :

$$\text{Rank (T) + Nulity (T) = Dim U}$$

3. Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3. Express the each standard basis vector in the linear combination of the vectors of $\alpha_1, \alpha_2, \alpha_3$.

4. For a matrix, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$, find a matrix P, such that $P^{-1} AP$ is a diagonal matrix.

5. Prove that every finite-dimensional inner product space has an orthonormal basis (Gram-Schmidt orthogonalisation process).

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that in any vector space number of elements in the basis is same.

2. Consider the vector space $P_2(t)$ of polynomials of degree ≤ 2. The polynomials $f_1(t) = t + 1$, $f_2(t) = t - 1$, $f_3(t) = (t - 1)^2$ form a basis S of $P_2(t)$. Find the coordinates.

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & -1 & 2 \\ -1 & -2 & 5 & 4 \end{bmatrix}$$

3. Find the rank and nullity of the matrix

4. If V(F) be a finite dimensional vector space and V(F) is a direct sum of two subspaces W_1 and W_2, then prove that :

$$\dim V = \dim W_1 + \dim W_2$$

5. If E_1 and E_2 are projections on V such that $E_1E_2 = E_2E_1$. Then prove that E_1E_2 and $E_1 + E_2 - E_1E_2$ are projections.

6. If α, β are vectors in an inner product space V, then prove the triangular inequality i.e.,

$$\| \alpha + \beta \| \leq \| \alpha \| + \| \beta \|$$

7. Prove the necessary and sufficient condition that a linear transformation T on an inner product space V be $\hat{0}$ is that :

$$\langle T\alpha, \beta \rangle = 0 \ \forall \ \alpha, \beta \in V)$$

8. If f be the bilinear form on R^2 defined by :

$$f((x_1, y_1), (x_2, y_2)) = x_1 y_1 + x_2 y_2$$

then find the matrix of f in the ordered basis $B = \{(1, -1), (1, 1)\}$ of R^2.

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