A–0683

Total Pages : 4 **Roll No. ....**

MAT-504

Mathematics (MSCMAT/MAMT)

(Advanced Differential Equation–I)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (a) Solve :

$$(D^2 - 4D + 4)\, y = x^2 + e^x + \cos 2x$$

(b) Solve :

$$(D^2 - 5D + 6)\, y = x\, e^{4x}$$

2. Derive Abel's formula for the Wronskian of two solutions of a second-order linear differential equation.

3. State and prove Picard's theorem.

4. Explain the concept of orthogonal trajectories and derive their general solution.

5. To show that :

(a) $$\int_{-1}^{1} \frac{T_r(x) T_n(x)}{\sqrt{1-x^2}}\, dx = \begin{cases} 0, & r \neq n \\ \frac{\pi}{2}, & r = n \neq 0 \\ \pi & r = n = 0 \end{cases}$$

(b) $$\int_{-1}^{1} \frac{U_r(x) U_n(x)}{\sqrt{1-x^2}}\, dx = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & r = n \neq 0 \\ \pi & r = n = 0 \end{cases}$$

Section–B

(Short Answer Type Questions) $4\times8=32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Solve :

   (a) $xdy - ydx = \sqrt{x^2 + y^2}$

   (b) $(1 + e^{x/y})\, dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

2. Solve by the method of Variation of parameters :

$$\frac{d^2y}{dx^2} + n^2 y = \sec nx$$

3. Prove that limit cycle is a closed curve.

4. Define the following :

   (a) Periodic Solution

   (b) Liapunov function

   (c) Singular points

   (d) Linear Differential Equation

5. Using the method by Krylov and Bogoliubov, solve the differential equation :

$$\frac{d^2y}{dt^2} + y + \in \left(\frac{dy}{dt}\right)^2 = 0$$

6. Show that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}$$

7. Prove that :

(a) $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x}$

(b) $H_n''(x) = 4n(n-1)\,H_{n-2}(x)$

8. If $f(x)$ be a polynomials of degree m, prove that $f(x)$ may be expressed in form :

$$f(x) = \sum_{r=0}^{m} C_r L_r(x)$$

where $C_r = \int_0^\infty e^{-x} L_r(x)\,dx$.

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