**A–0681**

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# MAT-502

## Mathematics (MSCMAT/MAMT)

### (Real Analysis)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

***Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

**Section–A**

**(Long Answer Type Questions)** 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that every bounded sequence of real numbers has a convergent subsequence, also find limit superior, limit inferior of sequence $\{\sin n : n \in \mathbb{N}\}$.

2. State and prove Darboux's theorem.

3. A bounded function is integrable on $[a, b]$ iff for every there exists a partition of $[a, b]$ such that :

$$U(p, f) - L(p, f) < \in.$$

4. Prove that the union of a finite collection of measurable sets is measurable.

5. Let $(X, d)$ be a metric space and let $x$, $y$ and $z$ be any points of X. Then prove that :

$$d(x, y) \geq | d(x, z) - (z, y) |.$$

**Section–B**

**(Short Answer Type Questions)** 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Set of irrational number is countable.

2. Prove that sequence can have at most one limit.

3. Prove that the sequence $\{(-1)^n)$ is divergent.

4. A function is continuous on $[a, b]$, then there exists a number $k$ in $[a, b]$ such that :

$$\int_a^b f dx = f(k)(b-a)$$

5. Examine the Convergence of $\int_0^1 \frac{1}{x^3} dx$ .

6. Let $f_n(x) = \frac{x}{n+x^2}, x \in [0,1]$. Then show that sequence of function $(f_n\ (x)\}$ is uniformly convergent on $[0, 1]$.

7. Define Lebegue outer measure with example.

8. Prove that with $|\ x\ |$ the absolute value of the difference $x - y$, for each $x, y \in \mathbb{R}$, $(\mathbb{R}, d)$ is a metric space.

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