A–0680

Total Pages : 4 **Roll No. ....**

MAT-501

Mathematics (MSCMAT/MAMT)

(Advanced Abstract Algebra)

Examination, June 2025

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) $2\times19=38$

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If H is subgroup of G and let

$$N(H) = \{x \in G : xhx^{-1} = H\}$$

then show that :

(i) N(H) is the largest subgroup of G in which H is normal.

(ii) H is normal in G iff N(H) = G.

2. If G be the finite group and Z(G) be the center of the group G. Then prove that class equation of G can be written as :

$$O(G) = O[Z(G)] + \sum_{a \notin Z(G)} \frac{O(G)}{O[N(a)]}$$

Here, summation runs over one element a in each conjugate class containing more than one element.

3. Show that the collection of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers is a field.

4. Show that following polynomials are irreducible in the ring Z[X] of polynomials in one variable with integer coefficients.

(i) $f(x) = x^2 - 5$

(ii) $f(x) = 1 + (x+1) + (x+1)^2 + (x+1)^3 + (x+1)^4$

(iii) $f(x) = x^4 + x^3 + x^2 + x + 1$

5. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G (Fundamental theorem of group homomorphism).

Section–B

(Short Answer Type Questions) $4 \times 8 = 32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Determine the coset decomposition of the subgroup $H = \{I, (12)\}$ corresponding to the symmetric group S_3.

2. Prove that G/Z(G) is cyclic if and only if G is abelian.

3. Find the number of cycle in S_{10} which commutes with $\alpha = (543)\ (26)\ (78910)$.

4. If P be an odd prime and G is a group of order 2P then either $G \cong Z_{2P}$ or $G \cong D_P$.

5. H is maximal normal subgroup of G iff G/H is simple.

6. Every subgroup of a nilpotent group is nilpotent.

7. Prove that intersection of two subring is again a subring.

8. Prove that $(I_5, +_5, \times_5)$ is a field.

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