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MT-609

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Integral Equations)

4th Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, with the initial conditions $y(0) = 1$, $y'(0) = 0$.

2. Find the eigen values and eigen functions of the homogeneous integral equation $g(x) = \lambda \int_0^1 exetg\,(t)\,dt$ and solve it.

3. Solve :

$$g(x) = \cos x + \lambda \int_0^{\pi} \sin\,(x - t)\; g(t)\; dt$$

4. Solve the integral equation :

$$g(x) = e^x - \frac{e}{2} + \frac{1}{2} + \frac{1}{2}\int_0^1 g(t)\; dt$$

5. Solve the following Volterra integral equation of the second kind :

$$g(x) = x^2 + \int_0^x \sin(x - t)\; g(t)\; dt$$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that function $g(x) = xe^{-x}$ is a solution of the equation :

$$g(x) - 4\int_0^\infty e^{-(x+t)} g(t)\, dt = (x-1)\, e^{-x}$$

2. $g(x) = f(x) + \lambda \int_0^1 xtg(t)\, dt$.

3. Prove that the Kernels K$(x, t) = xt$ and L$(x, t) = x^2t^2$ are orthogonal on [–1, 1].

4. Find the Resolvent Kernel for Volterra integral equation with the Kernel K(x, t) = 1.

5. Find the Resolvent Kernel of the integral equation with the Kernel K$(x, t) = 2x$. Here $\lambda = 1$.

6. Solve the homogeneous Fredholm integral equation :

$$g(x) = \lambda \int_0^1 e^{(x+t)} g(t) dt$$

7. Solve the integral equation :

$$g(x) = \sin x + \lambda \int_4^{10} xg(t)\, dt$$

8. Write a short note on the various applications of integral equations.

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