A–121

Total Pages No. : 3] **[Roll No. ....**

MT-502

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Real Analysis)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) **2×19=38**

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. If E is a countable set, then $m^*(E) = 0$.

2. Proof that if a function is summable on E, then it is finite almost every where on E.

3. State and proof the Riesz – Fisher theorem.

4. Define the following :

   (i) Countable set

   (ii) Cntor set

   (iii) Algebra of sets

   (iv) σ–Algebra

5. If \$ is a semi ring then prove the following statement :

   (i) If $A \in \$$ and $A_1, A_2, \ldots\ldots A_n \in \$$, then $A - U_{i=1}^{n} A_i$ is a finite union of pairwise disjoint sets of \$ (and hence is a σ-set).

   (ii) For every $\{A_n\}$ of pairwise disjoint members of \$, the set $A - U_{i=1}^{n} A_i$ is a σ-set.

   (iii) Countable union and finite intersection of σ-set are σ-sets.

Section–B

(Short Answer Type Questions) 4×8=32

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define the following term :

   (i) Lebesguc measure of a set.

   (ii) Minkowski inequality

2. Prove that the closed interval [0, 1] is uncountable.

3. Give an example to show that the function $|f|$ is measurable but f is not measurable.

4. Show by an example that the Iebesgue integral of a nowhere zero function can be zero.

5. State and proof Fatou's Lemma.

6. An orthonormal system $\{\phi_i\}$ is complete iff it is closed.

7. Show that the L^p–space is a normed metric space.

8. State the following :

   (i) Almost everywhere property.

   (ii) The space L_2 of square summable functions.

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