A–120

Total Pages No. : 4] **[Roll No. ....**

MT-501

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Advanced Algebra–I)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (i) Define with example :

   (a) Homomorphism

   (b) Isomorphism

   (ii) Prove that any two conjugate class of a group are either disjoint or identical

2. (i) Prove that every finite abelian group is solvable.

   (ii) Prove that any two composition series for a group G are equivalent.

3. Let R be a ring and M be an R-module. Then :

   (i) $r0 = 0$ for all R

   (ii) $(-r)m = -(rm) = r(-m)$ for all $r \in$ R and $m \in$ M

   (iii) $r(m - n) = rm - rn$ for all $r \in$ R and $m \in$ M

   (iv) $(r - s) = rm - sm$ for all $r \in$ R and $m \in$ M

4. If K is a finite field extension of a Field F and L is a finite field extension of K, then L is a finite field extension of F and [L : F] = [L : K] [K : F].

5. Let G_1 and G_2 be two groups. Then prove that $G_1 \times G_2$ is a group.

Section–B

(Short Answer Type Questions) **4×8=32**

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. If $B = \{e_1 = (1, 0), e_2 = (0, 1)\}$ is the usual basis of $\mathbb{R}^2$. Determine its dual basis.

2. Prove that every ring of polynomial $F[x]$ over a field F is a Euclidean ring.

3. Prove that the following mapping is linear $t : \mathbb{R}^3 - \mathbb{R}^3$ given by :

$$t(x, y, z) = (z, x + y) \text{ for all } (x, y, z) \in \mathbb{R}^3$$

4. Let G_1 and G_2 be two groups. Prove that :

$$G_1 \times G_2 \cong G_2 \times G_1$$

5. Define the following :

   (a) Prime element

   (b) Divisor

   (c) Euclidean ring

6. Let G be a finite group and $b \in$ G. Then prove that the number of elements conjugate to 'a' in G is equal to the index of the normalize of a in G.

7. Define the following :

   (a) Quotient Module

   (b) Module Homomorphism

8. Let Q be the field of rational numbers, then show that :

$$Q[\sqrt{2}, \sqrt{3}] = Q[\sqrt{2} + \sqrt{3}]$$

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