**A-133**

**Total Pages : 4** **Roll No. ........................**

# MT–604

## M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

### (Integral Transforms)

I[st] Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

***Note* :–** This paper is of Seventy (70) marks divided into two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

### Section–A

**(Long Answer Type Questions)** 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Find the laplace transform of :

(a) $\cosh^2 4t$

(b) $t^2 e^t \sin 4t$

(c) $(1 + te^{-t})^3$

2. Use complex inversion formula to obtain the inverse Laplace transform of :

$$\frac{p}{(p+1)(p-1)^2}$$

3. A flexible string has its end points on the $x$-axis at $x = 0$ and $x = c$ At time = 0, the string is given a shape defined by $b\sin\left(\frac{\pi x}{c}\right), 0 < x < c$ and released. Find the displacement of any point $x$ of the string at any time $t > 0$.

4. State and Prove Inversion formula for Hankel transform.

5. Solve $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, x > 0, t > 0$ subject to conditions :

(i) $U(0, t) = 0$

(ii) $U = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$

(iii) $U(x, t)$ is bounded.

## Section–B

**(Short Answer Type Questions)** $4\times8=32$

***Note :–*** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that :

$$L\left[\frac{\sin^2 t}{t};p\right]=\frac{1}{4}\log\left(\frac{p^2+4}{p^2}\right)$$

2. If $L^{-1}\left[\frac{p^2-1}{(p^2+1)^2};t\right]=t\cos t,$ then find

$$L^{-1}\left[\frac{(9p^2-1)}{(9p^2+1)^2};t\right].$$

3. Solve $(D^2+9)\,y=\cos 2t$, if $y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1$.

4. Find the Fourier sine and cosine transform of $f(t)$, if :

$$\begin{cases} t & ; \quad 0<t<1 \\ 2-t & ; \quad 1<t<2 \\ 0 & ; \quad t>2 \end{cases}$$

5. Find the Fourier cosine transform of $e^{-t^2}$.

6. Prove that :

$$M\{(1+x)^{-a}; p\} = \frac{\Gamma(a)\,\Gamma(a-p)}{\Gamma(a)}, \; 0 < \mathrm{Re}(p) < \mathrm{Re}(a)$$

7. Find the hankel transform of (i) $\frac{\cos ax}{x}$ (ii) $\frac{\sin ax}{x}$ taking $xJ_0(px)$ as the kernel.

8. Prove that :

$$M\{f(x); p\} = F(p), \text{ then } M\left[\frac{1}{x} f\left(\frac{1}{x}\right); p\right] = F(1-p)$$

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