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MT–601

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Analysis and Advanced Calculus–I)

3rd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections ‘A’ and ‘B’. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section ‘A’ contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Show that l^p is a Banach space for $1 \leq p < \infty$.

2. Let N be a non-zero normed linear space and :

$$M := \{x : x \in N \text{ and } \| x \| = 1\}$$

Then show that N is Banach space if and only if M is complete.

3. Show that Hilbert space is finite dimensional if and only if every complete set is basis.

4. State and prove the open mapping theorem.

5. Let $\{x_1, x_2, ..., x_n\}$ be a finite orthonormal set in a Hilbert space H. Show that for any :

$$x \in H, \sum_{i=1}^{n} |\langle x, x_i \rangle|^2 \leq \| x \|^2$$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that an orthogonal set of non-zero vectors is linearly independent.

2. Define a normed space. Is every metric induced by a norm ? Justify your answer.

3. Define Hilbert space, and Banach space. Give two examples of both of them.

4. State and prove Pythagorus theorem for a Hilbert space.

5. Let M = $\{(n, n, 0); (n, 0, n); (0, n, n)\} \subset \mathbb{R}^3$, where n is your exam roll-number. What is $M^{\perp}$? Justify your answer.

6. Prove or disprove with details: Every norm in C[0, 1] is complete. Here C[0, 1] represents the space of all real-valued continuous functions defined on the interval [0, 1].

7. If Y is a proper dense subspace of a Banach space X, then show that Y is not a Banach space in the induced norm.

8. State the Riesz representation theorem. Show, by using the Riesz representation theorem, that the dual of l^2 is l^2.

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