A–129

Total Pages : 5 **Roll No. ....**

MT–510

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Mechanics–II)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. A circular disc, of radius a, has a thin rod pushed through its centre perpendicular to its, plane, the length of the rod being equal to the radius of the disc. Show that the system can not spin with the rod vertical unless the angular velocity is greater than :

$$\sqrt{\frac{20g}{a}}$$

2. A particle of unit mass moves along OX under a constant force f starting from rest at the origin at time $t = 0$. If T and V are the kinetic and potential energies of the particle, calculate :

$$\int_0^{t_0} (\mathrm{T} - \mathrm{V})dt$$

Evaluate this for the varied motion in which the position of particle is given by :

$$x = \frac{1}{2} ft^2 + \in ft(t - t_0)$$

Where $\in$ is a constant; and show that the result is in agreement with Hamilton's principle. What are the essential features of the varied motion that ensure this agreement ?

3. Find the equation of continuity in Lagrange's method. Show that it is equivalent to :

$$\frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$$

4. Derive Euler's dynamical equations of motion in vector notation.

5. Derive equations of motion under impulsive force in Cartesian form.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. For a two-dimensional flow the velocities at a point in the fluid may be expressed in the Eulerian coordinates by :

$$u = x + y + 2t \text{ and } v = 2y + t$$

Determine the Lagrange coordinate as function of the initial position x_0 and Y_0 and the time t.

2. If the velocity of an incompressible fluid at the point (x, y, z) is given by :

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$$

Prove that the liquid motion is possible and the velocity potential is :

$$\left(\frac{\cos\theta}{r^2}\right)$$

3. Show that :

$$\frac{x^2}{a^2}\sin^2 t + \frac{y^2}{b^2}\cos ec^2 t = 1$$

is a possible form for the boundary surface of a liquid.

4. Write down the Bernoulli's equation for the unsteady, irrotational motion of an incompressible fluid.

5. Determine the image of the doublet with respect to a straight line.

6. Write the complex potential due to a doublet which makes an angle α with x-axis.

7. If there are sources at $(a, 0)$, $(-a, 0)$ and sinks at $(0, a)$, $(0, -a)$ all of equal strength. Show that the circle through these four points is a stream line.

8. Establish the relation between the Lagrangian and Eulerian method.

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