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MT–509

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Differential Geometry and Tensor–II)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and Prove Gauss-Bonet Theorem.

2. State and Prove Gauss's Characteristic equation.

3. Define :

   (a) Mixed tensor of second order

   (b) Symmetric tensor

   (c) Skew Symmetric tensor

   (d) conjugate symmetric tensor

4. Show that the metric of Euclidean space refered to cylindrical coordinates is given by :

$$(ds)^2 = (dr)^2 + (rd\theta)^2 + (dz)^2$$

Determine its metric tensor and conjugate metric tensor.

5. Prove that the integral :

$$\int_{t_0}^{t_1} f(x^i, \dot{x}^i)dt$$

has stationary value on the curve whose differential equations are :

$$\frac{\partial f}{\partial x^i} = -\frac{d}{dt}\left(\frac{\partial f}{\partial \dot{x}^i}\right) = 0 \text{ where } \dot{x}^i = \frac{dx^i}{dt}$$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that the curves $u + v$ = constant are geodesics on a surface with metric $(1 + u^2)du^2 - 2uv$ and $v + (1 + v^2)dv^2$.

2. Prove that geodesic curvature vector of any curve is orthogonal to curve.

3. For any surface prove that :

$$\frac{\partial}{\partial u}(\log H) = 1 + \mu \text{ and } \frac{\partial}{\partial v}(\log H) = m + v$$

where u and v are parameters and symbols have their usual meaning.

4. Prove that the law of transformation for a covariant vector is transitive.

5. Define :

   (a) Conjugate metric tensor

   (b) Riemannian space

6. State and Prove Ricci's theorem.

7. Prove that covariant differentiation of invariants is commutative.

8. Define :

   (a) Bianchy identity

   (b) Ricci tensor

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