Total Pages : 4 **Roll No. ....**

MT–508

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Special Functions)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note :– This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note :– Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

in series.

2. Prove that :

(i) $\int_{-1}^{+1} P_m(x)P_n(x)dx = 0 \quad \text{if } m \neq n$

(ii) $\int_{-1}^{+1} (P_n(x))^2 dx = \frac{2}{2n+1} \quad \text{if } m = n$

3. Prove that :

(i) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

(ii) $xJ_n'(x) = xJ_{n-1}(x) - nJ_n(x)$

4. Prove that :

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x)H_m(x)dx = \begin{cases} 2^n n!\sqrt{\pi} & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

5. Prove that :

(i) $L_n(0) = 1$

(ii) $L_n'(0) = -n$

(iii) $L_n''(0) = \frac{n(n-1)}{2}$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Define :

   (a) Ordinary point and singular point

   (b) Irregular singular point

   (c) Radius of convergence

2. Prove that :

$$ {}_1F_1(a, b; z) = \frac{1}{(a)} \int_0^{\infty} e^{-t} t^{a-1} {}_0F_1(-; b; zt) dt $$

3. Prove that :

$$ (2n + 1)(1 - x^2)Q'_n = n(n + 1)(Q_{n-1} - Q_{n+1}) $$

4. Prove that :

$$ J_0(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x \sin \varphi) d\varphi = \frac{1}{\pi} \int_0^{\pi} \cos(x \cos \varphi) d\varphi $$

5. Prove that :

$$ H''_n(x) = 4n(n - 1)H_{n-2}(x) $$

6. Prove that :

$$L_{n-1}^{k}(x) + L_{n}^{k-1}(x) = L_{n}^{k}(x)$$

7. State and Prove Gauss's theorem.

8. If c is neither zero nor negative integer, then :

$$_1F_1(a; c; z) = e^z \, _1F_1(c - a; c - z)$$

****