A–126

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MT–507

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Topology)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. What is a subbase for a topology ? Let (X, J) be a topological space and δ be a family of subsets of X. Show that δ is a subbase for J if and only if J is the smallest topology containing δ. Furthermore, find a subbase δ for the usual topology J_u on $\mathbb{R}$ with the property that δ is not a base for J_u.

2. Define a continuous map between two topological spaces. Let $f : X \to Y$ be a continuous surjection. If A is a dense subset of X, then show that $f(A)$ is dense subset of Y.

3. Show that a topological space X is T_3 if and only if for every open set $G \subset X$ and a point $x \in G$, there exists and open set $H \subset X$ such that $x \in H \subset \bar{H} \subset G$.

4. Determine which of the following sets are either open or closed in their respective topologies. Give proper justification in support of your answer.

   (i) Countable subsets of a co-countable topology.

   (ii) [0,1) in the indiscrete topology on $\mathbb{R}$

   (iii) $\left\{\left(\sqrt{(2)}, \sqrt{(3)}\right) \cap Q\right\}$ in the subspace topology on Q induced by the usual topology on R.

5. LetX be a Hausdorff space and let A be a compact subset of X. Then show that for each $x \in X \setminus A$, there exist disjoint open neighbourhoods of A and x. Furthermore, show that every compact subset of a Hausdorff space is closed.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. What is a Topology on a set X ? Let $X = \mathbb{N}$ the set of natural numbers, and let J consist of the empty set and all finite subsets of $\mathbb{N}$. Is J a topology on X. Justify your answer.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by :

$$f(x) = \begin{cases} 2x+1 & \text{if} \quad x \geq 0 \\ 2x+2 & \text{if} \quad x < 0 \end{cases}$$

In which of the following situations f is continuous. Justify your answer.

- $\mathbb{R}$ with usual topology on both domain and codomain.
- $\mathbb{R}$ with discrete topology on domain side and usual topology on codomain side.

3. Define T_3 and T_4 spaces. Show that a T_4 space need not be T_3.

4. Prove or disprove : Intersection of two compact sets is again compact.

5. Prove or disprove : Every discrete space X is locally disconnected.

6. Define product topology. Explain it with two examples one in finite product case and the other in infinite product case.

7. Define compactness. Show that continuous image of a compact set is compact.

8. Define connectedness. Show that continuous image of a connected set in connected.

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