A–124

MT–505

M.A./M.Sc. MATHEMATICS (MAMT/ MSCMT)

(Mechanics–I)

1st Semester Examination, 2024 (June)

Time : 2:00 Hrs.   Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions)   2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. A rod of length $2a$ revolves with uniform angular velocity w about a vertical axis through a smooth joint at one extremity of rod so that it describes a cone of semi-vertical angle α, show that :

$$w^2 = \frac{3g}{4a\cos\alpha}$$

Prove also that the direction of reaction at the hinge makes with the vertical an angle :

$$\tan^{-1}\left(\frac{3}{4}\tan\alpha\right)$$

2. Define centre of percussion. Find the centre of percussion of a rod suspended freely from one end struck by a blow.

3. Discuss the motion of a uniform sphere which rolls down an inclined plane, rough enough to prevent any slipping.

4. Derive Euler's geometrical equations of motion.

5. Two equal rods AB and BC , each of length l , smoothly jointed at B , are suspended from A and oscillates in a vertical plane through A. Show that the periods of normal oscillations are $\dfrac{2\pi}{n}$ where, :

$$n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$$

Section–B

(Short Answer Type Questions) 4×8=32

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. A uniform semi circular arc of mass m and radius a, is fixed at its ends to two point in the same vertical line, and is rotating with constant angular velocity w. Show that the horizontal thrust on the upper end is :

$$m.\frac{g+w^2a}{\pi}$$

2. Find the moment of the effective forces about the axis of rotation, when a rigid body is rotating about a fixed axis.

3 . Find the equation of motion of a simple pendulum with the help of Lagrange's equations.

4. Find the time of oscillation of a compound pendulum consisting of a rod of mass *m* and length *a* carrying at one and a sphere of mass *m* and diameter 2*b*, the other end of the rod being fixed.

5. Define the following :

    (i) Principle of conservation of angular momentum under finite forces

    (ii) Invariable Line

6. Show that in the free motion of body with an axis of symmetry (C) about its C.G. If n denotes the spin about the axis C and ϕ denotes the Euler's third angle then :

$$A\dot\phi = (A - C)n$$

7. Find the moment of momentum of a body about the fixed origin.

8. Find the position of centre of precussion in the following cases :

    (i) Uniform circular plane, axis is horizontal tangent.

    (ii) A uniform rod with one end fixed

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