A–122

Total Pages : 4 **Roll No. ....**

MT–503

M.A./M.Sc. MATHEMATICS (MAMT/MSCMT)

(Differential Equation and Calculus of Variation)

1st Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$x^2 y \frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y \right)^2 = 0$$

2. $q^2 r - 2pqs + p^2 t = 0$ by Monge's method.

3. Use the method of separation of variables to solve the PDE :

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

4. Find the eigenvalues and eigenfunction for the boundary value problem :

$$y'' - 4y' + (4 - 9\lambda)y = 0;\ y = (0),\ y(a) = 0$$

where 'a' is a positive real constant.

5. Find the convex curve of length L that encloses greatest possible area.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Solve :

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4\left(\frac{dy}{dx}\right)^3=0$$

2. Solve :

$$(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$$

3. Solve :

(i) $(2x + y^2 + 2xz)dx + 2xydy + x^2dz = dt$

(ii) $(xdx + ydy + zdz)^2z = \{(z^2x^2y^2)(xdx + ydy + zdz)dz\}$

4. Obtain the Euler's s-Lagrange equation for the extremals of the functional :

$$\int_{x_1}^{x_2}[y^2 - yy' + y'^2]dx$$

5. Solve :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$$

6. Solve :

(a) $rx = (n - 1)p$

(b) $2yq + y^2t = 1$

7. Find the characteristics of :

$$x^2r + 2xys + y^2t = 0$$

8. Define the following :

(a) Total differential equation

(b) Laplace equation

(c) Eigen value and eigen function

(d) Linear functional

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