A–084

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MSCPH–501

M.Sc. PHYSICS (MSCPH)

(Mathematical Physics)

1st Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into two (02) Sections 'A' and 'B'. Attempt the questions contained in these sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (a) State and prove Cauchy's integral theorem.

   (b) Evaluate the following integral using Cauchy's integral formula :

$$\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$$

   Where C is the circle $|z| = \frac{3}{2}$.

2. (a) From the law of transformation of a second rank covariant tensor :

$$A_{ij} = \frac{\partial \bar{x}^a}{\partial x^i}\frac{\partial \bar{x}^b}{\partial x^j}\bar{A}_{ab}$$

   Obtain the reverse transformation.

   (b) Define symmetric and anti-symmetric tensor.

3. (a) Describe linear ordinary differential equations of first and second order.

   (b) Explain heat equation in two and three dimension.

4. Describe recurrence formula for $H_n(x)$ and to show that $H_n(x)$ is a solution of Hermite equation. Also find the value of :

$$\int_{-\infty}^{\infty} e^{-x^2} H_2(x) H_3(x)\,dx$$

5. Show that the Laplace transform posses the properties of linearity, shifting and change of scale.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that Kronecker delta is a mixed tensor of rank 2.

2. State and explain Cayley-Hamilton theorem.

3. Solve Bessel's differential equation :

$$x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + (x^2 - n^2)y = 0$$

and determine its solution when x is an integer.

4. Prove the orthogonality of the Hermite Polynomials :

$$\int_{-\infty}^{\infty} e^{-x^2} \mathrm{H}_m(x)\mathrm{H}_n(x)dx = 0, \quad m \neq n$$

5. Show that the Hermite polynomials defined through a suitable generating function satisfy its differential equation.

6. Find the Fourier transform of the function :

$$f(t) = \begin{cases} 0 & t < 0; \\ e^{-\alpha t} & t \geq 0; \end{cases} \quad \propto > 0$$

7. Find the Laplace transform of the function $t.e^{2t}$.

8. Write Laguerre differential equation and related polynomials.

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