A–833

Total Pages : 5 **Roll No. ....**

MCS-501

MCA/MSCIT

(Discrete Mathematics)

1st/3rd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

(Long Answer Type Questions) 2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. (a) Define a partial order relation. Let X = (1, 2, 3, 4, 9, 12, 18, 24, 36} and R = {(x, y) : $x|y$, that is, x divides y, $\forall x, y \in$ X} be a partial order relation on X. Draw the Hasse diagram of the relation R. 10

(b) Define one-one onto function. Show that the function f : R $\rightarrow$ R defined as $f(x) = \frac{3}{2}(x-3)$ for all $x \in$ R is one to one onto. 9

2. (a) Define the logical connectives : Disjunction, Conjunction, Conditional, Bi-conditional and negation using suitable examples. 10

(b) Check the validity of the following argument :

"If I watch a movie, then study. If I play chess, then I study. Therefore, if I either watch a movie or play chess, then I study". 9

3. (a) Define principal of mathematical induction. Using mathematical induction prove that for every non-negative integer n, $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$. 10

(b) Solve the recurrence relation : 9

$$a_n - 3a_{n-1} + 2a_{n-2} = 3n + 2$$

4. (a) Discuss proof by contraposition. Using the method of proof by contraposition, prove that if n^2 is odd, then n is odd. 10

(b) Let (G, *) be a group. Prove the followings :

(i) The identity element is unique

(ii) $(a * b)^{-1} = b^{-1} * a^{-1}$ for $a, b \in$ G. 9

5. (a) Define a subgroup. Prove that the order of each subgroup of a finite group is a divisor of the order of the group. 10

(b) Define connected and disconnected graphs. Prove that if a graph G (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining the two vertices. 9

Section–B

(Short Answer Type Questions) 4×8=32

Note :– Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Draw the Venn diagram of the following sets :

   (i) $(X \cup Y) \cap Z$

   (ii) $Z \cap (X \cup Y)$

   (iii) $(X \cup Y) - Z$

   (iv) $X - (Y \cup Z)$

2. Write predicates for the following sentences :

   (i) All cities of Uttarakhand are not highly populated.

   (ii) Some of the integers are positive.

3. Define a tree. Prove that there are $n - 1$ edges in a tree with n vertices.

4. Let $\Sigma = [a, b]$, define Σ^* and design a DFA that accepts all the strings containing *bab* as a substring.

5. Define Mealy machine with the help of a suitable example.

6. Define a subgroup. Prove that the union of two subgroups is a subgroup if one of them is contained in the other.

7. Define generating function. Find the generating functions corresponding to the following numeric function :

$$a_n = \begin{cases} 2^n & \text{when } n \text{ is even} \\ -2^n & \text{when } n \text{ is odd} \end{cases}$$

8. What do you mean by graph coloring ? Define chromatic number and chromatic polynomial with the help of suitable examples.

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