A–110

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MAT-504

MATHEMATICS
(MSCMAT/MAMT)

(Advanced Differential Equation–I)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) **2×19=38**

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Solve :

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

by using variation of parameter method.

2. A body of mass 0.2 kg is hung from a spring of constant 80 N/m. The body is subjected to a resistive force given by bv, where v is the velocity in m/s. Calculate the value of the undamped frequency and the value of τ if the damped frequency is $\sqrt{3/2}$ of the undamped frequency.

3. Discuss the orthogonality property of Legendre polynomials.

4. State and prove Bessel differential equations.

5. Prove that the recurrence relations of the following :

   (a) $(n + 1)\, L_{n+1}(x) = (2n + 1 - x)\, L_n(x) - nL_{n-1}(x)$

   (b) $xL_n'(x) = nL_n(x) - nL_{n-1}(x)$

   (c) $L_n'(x) = -\sum_{r=0}^{n-1} L_r(x)$

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the differential equation of the family of curvesy = e^{mx}, where m is arbitrary constant.

2. Solve the following differential equations :

   (a) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

   (b) $\frac{dy}{dx} = \frac{\sin x + x \cos x}{y(2 \log y + 1)}$

3. Find the orthogonal trajectories of cardioids $r = a(1 + \cos \theta)$.

4. (a) Solve $(D^4 + 4) y = 0$.

   (b) Solve $(D^4 + D^2 + 1) y = 0$

5. Discuss the various types of critical points.

6. Discuss the importance of singular points.

7. Show that :

   (a) $L_n(0) = 1$

   (b) $L_n(0) = n!$

8. Prove that :

$$\frac{d}{dx}[xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$$

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