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MAT-503

MATHEMATICS
(MSCMAT/MAMT)

(Advanced Statistics)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) **2×19=38**

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of four balls transferred 3 are white and 1 is black ?

2. The diameter of an electric cable , say X, is assume to be a continuous random variable with probability density function :

$$f(x) = 6x(1 - x),\ 0 \leq x \leq 1$$

(i) Check that $f(x)$ is probability density function and

(ii) Determine a number b, such that for $0 \leq x \leq 1$, $f(x) \geq 0$.

3. Use Chebyshev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6.

4. In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible : Variance of X = 9. Regression equations : 8X – 10Y +

66 = 0, 40X – 18Y = 214. What were, (i) the mean value of X and Y (ii) the correlation coefficient between X and Y and (iii) the standard deviation of Y ?

5. Prove that for normal distribution, the mean deviation from the mean equals to $\frac{4}{5}$ of the standard deviation approximately.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Data on readership of a certain magazine show that the proportion of 'male' readers under 35 is 0.40 and over 35 is 0.20. If the proportion of readers under 35 is 0.70, find the proportion of subscribers that are "females over 35 years". Also calculate the probability that a randomly selected male subscriber is under 35 years of age.

2. Let X be a discrete random variable having probability mass function :

$$px(x) = \begin{cases} 3/4 \text{ if } x = 1 \\ 1/4 \text{ if } x = 2 \\ 0 \text{ otherwise} \end{cases}$$

Find the third central moment of X.

3. A binomial variable X satisfies the relation $9P(X = 4) = P(X = 2)$ when $n = 6$. Find the value of the parameter p and $P(X = 1)$.

4. In a sample of 1000 cases, the mean of a certain test is 14 and S.D. is 2.5. Assuming the distribution to be normal, find

   (i) How many students score between 12 and 15 ?

   (ii) How many score above 18 ?

   (iii) How many score below 8 ?

   (iv) How many score 16 ?

5. Obtain the rank correlation coefficient for the following data :

X	Y
68	62
64	58
75	68
50	45
64	81
80	60
75	68
40	48
55	50
64	70

6. To sample polls of votes for two candidates A and B for a public office are taken, one from among the residents of rural areas. The results are given in the table. Examine whether the nature of the area is related to voting preference in this election.

Votes for Area	**A**	**B**	**Total**
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

7. Consider a random sample X_1, X_2, ..., X_n from a population which follows Bernoulli distribution with parameter p (i.e. if X is population variate, then X take the value 1 with probability p and the take the value 0 with probability $(1 - p)$. Let :

$$\text{Let } t = \frac{\sum_{i=1}^{n} x_i}{n} \times \left(1 - \frac{\sum_{i=1}^{n} x_i}{n} \right)$$

Then t is a consistent estimator of $p(1 - p)$.

8. What are the criticisms against the use of Baye's Theorem in probability theory ?