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MAT-502

MATHEMATICS
(MSCMAT/MAMT)

(Advanced Real Analysis)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that a sequence of real numbers converges iff it is Cauchy.

2. Consider I = $[a, b]$ be a closed bounded interval and $f : I \to \mathbb{R}$ be continuous on I. Then prove that f has an absolute maximum and an absolute minimum point on I.

3. Prove that a bounded function f is integrable on $[a, b]$ iff for every $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

4. Show that the series $\sum_{n=1}^{\infty} \frac{x}{1+nx^2}$ is uniformly convergent for all real x.

5. Prove that Intersection of finite number of open set is open in metric space.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let $\mathbb{Z}$ be the set of all integers then prove that $\mathbb{Z}$ is countable.

2. State and prove Darboux's Theorem.

3. Prove that if $f : [a, b] \to \mathbb{R}$ be a bounded function and α be a monotonically increasing function on $[a, b]$. Consider P be any Partition of $[a, b]$. Then $U(P, f, \alpha)$ and $L(P, f, h)$arebounded.

4. Test for convergence :

$$\int_0^{\pi/2} \sin^{p-1} x \cos^{q-1} x \, dx$$

5. Prove that any countable set is measurable.

6. Prove that the sequence $\{x_n\}$ where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n'}$$

does not satisfy Cauchy's criterion of convergence.

7. Prove that let (X, d) be a metric space and let X_1 be a connected subset of X. If X_2 is a subset of X such that $X_1 \subseteq X_2 \subseteq \overline{X_1}$, then X_2 is connected.

8. Define fixed point, state and prove Banach Contraction principle.

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