A–107

Total Pages No. : 3] **[Roll No. ....**

MAT-501

MATHEMATICS
(MSCMAT/MAMT)

(Advanced Abstract Algebra)

Ist Semester Examination 2024 (June)

Time : 2 : 00 Hours] [**Max. Marks :** 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

(Long Answer Type Questions) 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that the set of all distinct cosets of normal subgroup of a group is a group with respect to composition multiplication of cosets.

2. State and prove the necessary and sufficient condition for a non-empty subset of a ring to be a subring of the ring.

3. Prove that every finite integral domain is a field.

4. Evaluate all permutations in A_5 which commutes with :

   (i) $\alpha = (12345)$

   (ii) $\beta = (123)$

   (iii) $\gamma = (12)(34)$

5. State and prove the Jordan-Holder theorem.

Section–B

(Short Answer Type Questions) 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that the alternating subgroup A_n is the normal subgroup of the symmetric group S_n.

2. Prove that conjugacy is an equivalence relation on G .

3. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G (First fundamental theorem on group homomorphism).

4. Write the class equation of the quaternion group :

$$Q_4 = \{\pm 1, \pm i, \pm j, \pm k\}$$

5. Give atleast two exahiples to prove that the product of two cyclic group may or may not be cyclic.

6. Prove that $(I_3, +_3, \times_3)$ is a field.

7. Prove that each finite *p*-group is nilpotent.

8. State and prove Eisenstein's criterion for irreducibility of polynomials over Q.

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