A–896

Total Pages : 4 **Roll No. .....**

MAT–509

M.Sc. MATHEMATICS (MSCMT)

(Mathematical Methods)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Obtain the Fourier series to represent $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

2. Find the Fourier series to represent the function $f(x)$ given by :

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases},$$

and hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$$

3. Find the extremal of the functional :

$$I[y(x), z(x)] = \int_a^b (2zy - 2y^2 + y'^2 - z'^2)dx$$

4. Find the eigenvalues and the corresponding eigenfunctions of the integral equation :

$$y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t)dt$$

5. State and prove necessary condition for the existence of extremal. Find the extremals of the functional :

$$I[y(x)] = \int_a^b F(y'')^2 - 2(y')^2 + y^2]dx$$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Laplace transform of the function $L\{\sin^2 t\}$.

2. Extremize :

$$I[y(x)] = \int_0^1 \left(\sqrt{1+y'^2} \right) dx, \text{ with } y(0) = 0, y(1) = 1$$

3. Solve the Fredholm integral equation :

$$y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t) dt$$

4. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

5. Solve the Poisson's equation $u_{xx} + u_{yy} = -1$.

6. Find the modified Green's function for the system :

$$y'' = 0, -1 < x < 1.$$

Subject to the conditions :

$$y(-1) = y(1) \text{ and } y'(-l) = y'(1)$$

7. Find the iterated kernel for the following kernel :

$$k(x, t) = e^x \cos t,\ 0 < x < 2\pi;\ a = 0,\ b = \pi$$

8. Find the eigenvalue and the corresponding eigenfunctions of the homogeneous integral equation :

$$y(x) = \lambda \int_0^1 \sin \pi x \cos \pi x\, y(t) dt$$

****