A–894

Total Pages : 4 **Roll No. ....**

MAT–507

M.Sc. MATHEMATICS (MSCMT)

(Measure Theory)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and proof Lebesgue Dominated Convergence Theorem.

2. Let f be a nonnegative measurable function on E. Then $\int_E f = 0$ if and only if $f = 0$ almost everywhere. Proof that.

3. Let v be a signed measure on (X, M} and $E \in M$ where $0 < v(E) < \infty$. Then there exists a positive set set $A \subset E$ with positive measure. Proof that.

4. Define the following term :

   (a) Boolean ring

   (b) σ-ring

   (c) Boolean algebra

   (d) σ-algebra of sets

   (e) Outer Measure

   (f) Measurable sets

   (g) Lebesgue Measure

   (h) Lebesgue Outer Measure

   (i) The Cantor set

5. State and proof that The Stone Weierstrass Approximation Theorem.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that the set Q of rational numbers is countably infinite.

2. Let X be an infinite set and B be the collection of all subsets A of X such that either A or A^c is finite. Show that B is an algebra but is not a σ-algebra.

3. Proof that A countable set has Lebesgue outer measure zero.

4. Proof that Every interval is measurable.

5. Show that the algebra generated by the set $(1, x^2\}$ is dense in C[0, 1] but fails to be dense in C[–1, 1].

6. State the following :

    (a) Almost everywhere (a.e.)

    (b) The Fatou's Lemma

    (c) Dini's theorem

    (d) Measurable function

7. Proof that the union of a countable collection of sets of measure zero is a set of measure zero.

8. Let E be a set of measure zero. Show that if f is bounded function on E and $\int_E f = 0$.

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