A–893

Total Pages : 3 **Roll No. ....**

MAT–506

M.Sc. MATHEMATICS (MSCMT)

(Topology)

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2\times19=38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and proof Tietze extension theorem ?

2. Consider X = Set of real numbers and collection $\mathbf{B} = \{(a, b) \subseteq R | a < b\}$. Show that B is a basis for any topology of set of real number ?

3. Proof that every second countable topological space (X, **J**) is a separable space.

4. Let X be an ordered set in the order topology, let Y be a subset of X that is convex in X. Then the order topology on Y is the same as the topology Y inherits as a subspace of X. Proof that.

5. Define the following term :

   (i) Topology on a set X

   (ii) Basis for a topology

   (iii) Topology **J** generated by **B**.

   (iv) Standard Topology

   (v) Interior of a set.

   (vi) Closure of a set

   (vii) Limit point of set

   (viii) If $(X, \mathbf{J}_x)$ and $(Y, \mathbf{J}_y)$ be topological spaces. Define Continuity of a function $f : X \to Y$

   (ix) Homomorphism

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let $\mathbf{J}_1 = (\phi, \{l\}, X_1\}$ be a topology on $X_1 = \{1, 2, 3\}$ and $J_2 = \{\phi, X_2, \{a\}, \{b\} \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ be a topology for $X_2 = \{a, b, c, d\}$. Find a base for the product topology **J** ?
2. In any topological space, prove that $\overline{A} = A \cup D(A)$.
3. Proof that every regular topological space (X, **J**) is a Hausdorff space.
4. State and proof Baire category theorem.
5. Proof that a topological space (X, **J**} is compact if and only if every net in X has a subset that converges to an element in X.
6. Proof that the composite of two quotient maps is a quotient map.
7. Proof that Every closed subspace of a compact space is compact.
8. Proof that R with usual topology is first countable.

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