# A–892

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# MAT–505

## M.Sc. MATHEMATICS (MSCMT)

**(Advanced Linear Algebra)**

2nd Semester Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

***Note*** **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

### Section–A

**Long Answer Type Questions** $2\times19=38$

***Note*** **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Find the basis and dimension of null space and column space of the matrix (A) :

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

2. If $W_1$ and $W_2$ are finite-dimensional subspaces of a vector space V, then prove that, $W_1 + W_2$ is finite-dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

3. State and prove the rank nullity theorem.

4. Verify that the matrix :

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

is diagonalizabe.

5. State and prove the Cauchy-Schwartz inequality.

## Section–B

**Short Answer Type Questions** 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let $S = \{(x_1, x_2) : x_1, x_2 \in R\}$. For any $(x_1, x_2)$, $(y_1, y_2) \in S$ and scalar $c \in R$. Define vector addition and scalar multiplication as,

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 - y_2)$$

$$c(x_1, x_2) + (y_1, y_2) = (cx_1, cx_2)$$

Is S form a vector space (Verify that) ?

2. Let $P_3(R)$ be the vector space of set of polynomials of degree less than equal to 3 defined on R. Show that set $S = \{1 + x + x^2, 7 + x^3, 11 + x + x^2 + x^3, 13 + 4x\}$ are linearly independent.

3. Consider the vector space $P_2(t)$ of polynomials of degree $\leq 2$. The polynomials :

$$f_1 = t + 1, f_2 = t - 1, f_3 = (t - 1)2 = t_2 - 2t + 1$$

form a basis S of $P_2(t)$. Find the coordinates.

4. Verify that the mapping, $T : F^3 \rightarrow F^3$ defined by :

$$T(x, y, z) = (x - y + z, 2x + y - z, - x - 2y)$$

is a linear transformation.

5. Find the matrix of the linear transformation (T) on $V_3(R)$ with respect to the ordered basis :

$$B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$$

6. Let T be the linear operator on $R^3$ which is represented in the standard ordered basis by the matrix :

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

Find the minimal polynomial for T.

7. Let V be the vector space of all real polynomials of degree ≤2. Prove that :

$$< f(x), g(x) >= \int_0^1 f(x)g(x)dx, \forall f(x), g(x) \in V$$

is an inner product on V.

8. Every linear operator T on a finite dimensional complex inner product space V can be uniquely expressed as $T = T_1 + iT_2$, where $T_1$ & $T_2$ are self-adjoint linear operators on V.

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