A–900

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MAMT–09

INTEGRAL TRANSFORMS AND INTEGRAL EQUATIONS

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. With the help of Laplace transform solve the differential equation :

$$(D^2 + 4)y = f(t),\ y(0) = 0,\ y'(0) = 1$$

where $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$.

2. Find the solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $0 < x < 1$, $t > 0$ together with the conditions :

$$u(x, 0) = 3 \sin 2\pi x,\ u(0, t) = 0,\ u(1, t) = 0$$

3. Find $f(t)$, if it's Fourier sine transform is $\frac{p}{(1 + p^2)}$.

4. Prove that the characteristic numbers of a symmetric kernel are real.

5. Find the eigenvalues and eigenfunctions of the homogeneous integral equation :

$$g(x) = \lambda \int_0^1 K(x,t) g(t) dt,$$

where $K(x,t) = \begin{cases} x(t-1), 0 \le x \le t \\ t(x-1), t \le x \le 1 \end{cases}$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Find the Laplace transform of :

   (i) $e^{-2t} - \sin 5t + 4\cos 7t + 9t^3 - 5$

   (ii) $\cosh^2 4t$

2. Find the inverse Laplace transform of :

   (i) $\dfrac{p}{(p^2+a^2)^2}$

   (ii) $\dfrac{p+1}{(p^2+2p+2)^2}$

3. Find the Fourier sine and cosine transform of $f(t)$ if :

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

4. State the Mellin inversion theorem and convolution theorem for Mellin transform.

5. Define the following terms :

   (i) Integral equation

(ii) Linear and non integral equations

(iii) Singular integral equation

(iv) Convolution integral equation

6. Solve :

$$g(x) = e^x + \lambda \int_0^1 2e^x e^t g(t)dt$$

7. Solve :

$$g'(x) = x + \int_0^x g(x-t)\cos t\, dt, g(0) = 4$$

8. Proof that the eigenvalues of a symmetric kernel are real.

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