A–897

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MAMT–06

ANALYSIS AND ADVANCED CALCULUS

M.A./M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2024 (June)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Prove that every normed linear space is a metric space.

2. The real line space $\mathbb{R}$ and complex linear space $\mathbb{C}$ are Banach space under the norm $\| x \| = | x |, x \, \varepsilon \, \mathbb{C}$ or $\mathbb{R}$.

3. Define Hilbert space and consider the Banach space l_2^n consisting of all n tuples $x = (x_1, x_2, ..., x_n)$ of a complex number with norm $\| x \| = \left[\sum_{n=1}^{n} | x_i |^2\right]^{1/2}$ if $y = (y_1, y_2, y_n)$ and define inner product of x and y as $(x, y) = \sum_{i=1}^{n} x_i \bar{y}_i$, then prove that l_2^n is a Hilbert space.

4. Show that the function f, where

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if} \quad x^2+y^2 \neq 0 \\ 0 \quad , & \quad x = y = 0 \end{cases}$$

is continuous, possesses partial derivatives but is not differentiable at the origin.

5. Using Taylor's theorem expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. States and prove closed graph theorem.

2. Prove that every inner product space is a normed linear space.

3. Prove that an orthonormal set in a Hilbert space is linearly independent.

4. Prove that if T is an operator on a Hilbert space H, then T is normal $\Leftrightarrow$ its real and imaginary parts commute.

5. Show that the function $f(x, y) = |x| + |y|$ is continuous, but not differential at origin.

6. Examine the equality of f_{xy} and f_{yx}, where :

$$f(x, y) = x^3 y + e^{xy^2}$$

7. If $x = r \sin \theta \cos \phi$, $y = r \sin \phi$, $z = r \cos \theta$, then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$$

8. If T is an operator on a Hilbert space H and α, β are scalars such that $|\alpha| = |\beta|$, then $\alpha T + \beta T^*$ is normal

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