A–1049

Total Pages : 4 **Roll No. ....**

MT-609

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Integral Equations

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

Long Answer Type Questions 2×19=38

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Find the eigenvalues and eigen function of the homogeneous integral equation :

$$g(x) = \lambda \int_0^{\pi} [\cos^2 x \cos 2t + \cos 3x \cos^3 t] g(t) dt$$

2. With the aid of the resolvent kernel, find the solution of the integral equation :

$$g(x) = e^{x^2 + 2x} + 2\int_0^x e^{x^2 - t^2} g(t) dt$$

3. State and prove the Hilbert-Schmidt theorem.

4. Using the Fredholm determinants, find the resolvent kernel, when $K(x, t) = xe^t$; $a = 0$, $b = 1$.

5. Show that the integral equation :

$$g(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t) g(t) dt$$

possesses no solution for $f(x) = x$, but that it posseses infinitely many solution when $f(x) = 1$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that the function $g(x) = (1 + x^2)^{-3/2}$ is a solution of the Volterra integral equation :

$$g(x) = \frac{1}{(1+x^2)} - \int_0^x \frac{t}{1+x^2} g(t)dt$$

2. Form an integral equation corresponding to the differential equation :

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

with initial conditions $y(0) = 1$, $y'(0) = 0$.

3. If $y(x)$ is continuous function and satisfies the integral equation $y(x) = \lambda \int_0^1 K(x, t) y(t) dt$,

where

$$K(x, t) = \begin{cases} (1-t)x; & 0 \leq x \leq t \\ (1-x)t; & t \leq x \leq 1 \end{cases}$$

Then prove that $y(x)$ is also the solution of the boundary value problem :

$$\frac{d^2y}{dx^2} + \lambda y = 0; \ y(0) = 0, \ y(1) = 0$$

4. Find the solution of the integral equation :

$$g(x) = e^x + \lambda \int_0^1 2e^x e^t g(t) dt$$

5. Find the resolvent kernel of the kernel $K(x, t) = e^{x+t}$; $a = 0, b = 1$.

6. Prove that if a kernel is symmetric, then all its iterated kernels are also symmetric.

7. Using the Hilbert-Schmidt theorem, find the solution of the symmetric integral equation :

$$g(x) = x^2 + 1 + \frac{3}{2}\int_1^1 (xt + x^2t^2)g(t)dt$$

8. For the integral equation :

$$g(x) = f(x) + \lambda\int_a^b K(x,t)g(t)dt$$

Find $D(\lambda)$ and $D(x, t; \lambda)$ for the kernel $K(x, t) = \sin x$; $a = 0, b = \pi$.

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