A–1046

Total Pages : 4 **Roll No. ....**

MT-606

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Analysis and Advanced Calculus-II

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Show that the set of all normal operators on a Hilbert space H is a closed subset of β(H) which contains the set of all self-adjoint operators and is closed under scalar multiplication.

2. Prove that P is a projection on a Hilbert space H with range M and null space N, then $M \perp N$ iff P is a self adjoint, and in this case $N = M^{\perp}$.

3. Proof that if P_1, P_2, P_3 .... P_n subspaces are the projections on closed linear subspaces M_1, M_2, M_3 .... M_n of a Hilbert space H, then $P = P_1 + P_2 + P_3 + \ldots P_n$ is a projection if and only if the P_i's are pairwise orthogonal and then P is the projection on $M = M_1 + M_2 + M_3 + \ldots M_n$.

4. State Lipschitz's property and prove that if u be a non-negative continuous functions on an interval $\{0, c\}$, $(c > 0)$ satisfying the inequality $u(t) \leq at + k\int_0^t u(s)ds$ for all $t \in [0, c]$ then $u(t) \leq \frac{a}{k}(e^{kt} - 1)$ for $t \in [0, c]$.

5. Show that if T is an arbitrary operator on a finite dimensional Hilbert space H, then the eigenvalues of T constitute a non-empty finite subset of the complex plane. Furthermore, the number of points in this does not exceed the dimension n of the space H.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Show that if T be an operator on a Hilbert space H, then there exists a unique linear operator T* on H, such that $(Tx, y) = (x, T^*y)$ for all $x, y \in H$. Obviously T* is the adjoint operator H.

2. Show that if T be an operator on a Hilbert space H, then $(Tx, x) = 0$ for all $x \in H$ iff $T = 0$.

3. Define an adjoint operator and normal operator on a Hilbert space H and give an example.

4. Define orthogonal Projection, reducibility and Invariance of an operator on a Hilbert Space.

5. Define the definition of tangential of two function and derivative of a map with example where X and Y are Banach spaces.

6. Prove that if T be an arbitrary operator on a finite dimensional Hilbert space H, then the eigenvalues of T constitute a non empty finite subset of the complex plane. Furthermore, the number of points in this does not exceed the dimension n of the space H.

7. State and proof Taylor's theorem.

8. Proof that if X be a Banach space over the field K of scalars and let $f : [a, b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $\| Df(t) \| \leq Dg(t)$ at each point $t \in (a, b)$. Then $\| f(b) - f(a) \| \leq g(b) - g(a)$.

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