A–1040

Total Pages : 4 **Roll No. ....**

MT-510

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Mechanics-II

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. State and prove the Hamilton's principle and principle of least action.

2. If the velocity of an incompressible fluid at a point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$. Prove that the liquid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$.

3. Derive the equation of continuity in Cartesian coordinate system.

4. A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being π; show that, if the radius R of the sphere varies in any manner, the pressure at the surface of the sphere at any t time is :

$$\pi + \frac{\rho}{2}\left\{\frac{d^2}{dt^2}\mathrm{R}^2 + \left(\frac{d\mathrm{R}}{dt}\right)^2\right\}$$

5. What arrangement of sources and sinks will give rise to the function $w = \log\left(z - \frac{a^2}{z}\right)$? Draw a rough sketch of a stream line. Prove that two of the stream lines sub divided into the circle $r = a$ and axis of y.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. A particle of unit mass is projected so that its total energy is h in a field of force of which the potential energy is $\phi(r)$ at a distance r from the origin. Deduce from the principle of energy and least action that the differential equation of the path is :

$$c^2\left[r^2+\left(\frac{dr}{d\theta}\right)^2\right]=r^4[h-\phi(r)]$$

2. Find the equation of the stream lines passing through the point (1, 1, 1) for an incompressible flow $\vec{q}=2x\hat{i}-y\hat{j}-z\hat{k}$.

3. If $u = 2xy$ and $v = (a^2 + x^2 - y^2)$ are the velocity component of a fluid motion, then find the stream function.

4. Find the velocity vector $\vec{q}$ for the velocity potential $\phi = c(x^2 - y^2)$.

5. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial \theta} = 0$, where w be the angular velocity of a particle whose azimuthal angle is θ at time t.

6. State and prove the Bernoulli's theorem.

7. Derive the Cauchy-Riemann equations in polar coordinates.

8. State and Prove the Lagrange's equation of motion.

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