A–1036

Total Pages : 4 **Roll No. ....**

MT-506

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Advanced Algebra-II

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

Long Answer Type Questions $2 \times 19 = 38$

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Let F be a field then every polynomial of the positive degree in F[x] has splitting field.

2. Let $t : R^3 \to R^3$ be the linear transformation such that :

$$t(a, b, c) = (3a + c, -2a + b, -a + 2b + 4c)$$

What is the matrix of t in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$?

where $\alpha_1 = (1, 0, 1)$ $\alpha_2 = (-1, 2, 1)$, $\alpha_3 = (2, 1, 1)$

3. If M and N are subspaces of a finite dimensional inner product space V, then prove that :

$$(M + N)^{\perp} = M^{\perp} \cap N^{\perp}$$

4. If $\{u_1, u_2, \dots \dots, u_n\}$ is any finite orthonormal set is an inner product space V and v is any vector in V, then prove that :

$$\sum_{i=1}^{n} \left| \langle v, u_i \rangle \right|^2 \leq \| v \|^2$$

And equality holds if and only if v is in the subspace generated by $\{u_1, u_2, \dots \dots, u_n\}$.

5. Apply the Gram-Schmidt process to the vectors $v_1 = (1, 0, 1)$, $v_2 = (1, 0, -1)$ $v_3 = (0, 3, 4)$ to obtain an orthonormal basis for R^3 with the standard inner product.

Section–B

Short Answer Type Questions $4 \times 8 = 32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Let K be a field extension of a field F and $f(x) \in F[x]$ If $\phi : F \to K$ in an automorphism such that $\phi(a) = a \ \forall \ a \in F$ and $a \in K$ is a root of $f(x)$ then prove that $\phi(\alpha)$ is also root of $f(x)$.

2. For any matrix A over a field F. Prove that :

$$\text{rank (A)} = \text{rank }(A^T)$$

3. Let $t : R^2 \to R^2$ be a linear transformation defined by

$$t(a, b) = (2a - 3b, a + b) \ , \forall (a, b) \in R^2$$

Then find the matrix of t relative to the basis B = {(1, 0), (0, 1)} B′ = {(2, 3), (1, 2)}.

4. Let H be a subgroup of all automorphism of a field K. Then prove that fixed field of H is a subfield of K.

5. Let K = $Q\sqrt{2} = \left\{ a + \frac{b\sqrt{2}}{a}, b \in Q \right\}$ and F = Q. Find $G\left(\frac{K}{G}\right)$ and fixed field of $G\left(\frac{K}{F}\right)$.

6. Let A and B are two matrices over the same field F, the show that :

$$\det(A) = \det(B)$$

7. Prove that the eigenvalues of a self-adjoint linear transformation are real.

8. Define the following :

   (a) Galois Extension and Galois Group

   (b) Rank and Nullity matrix

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