A–1032

Total Pages : 3 **Roll No. ....**

MT-502

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Real Analysis

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions 2×19=38

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Let $\langle E_i \rangle$ be an infinite increasing sequence of measurable sets, that is $E_1 \subset E_2 \subset E_3$ .... . Then prove that :

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

2. Let f and g be two bounded measurable functions defined on a measurable set E. Then prove that $f \pm g$ is L-integrable over E and

$$\int_E (f \pm g)(x)dx = \int_E f(x)dx \pm \int_E g(x)dx$$

3. Let f and g be two summable functions on a set E and c be a constant, then prove that function c.f. is also summable on E and $\int_E (cf)(x)dx = c\int_E f(x)dx.$

4. State and prove the Riesz-Fisher theorem.

5. Prove that the space L^p is complete for $p \geq 1$.

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that the Cantor's set C is uncountable with outer measure zero.

2. If E_1 and E_2 are measurable sets then prove that $E_1 - E_2$ is measurable and for $E_1 \supset E_2$

$m^*(E_1 - E_2) = m^*(E_1) - m^*(E_2)$.

3. Prove that the characteristic function ϕ_A of a set A is measurable if and only if A is measurable.

4. Prove that every bounded measurable function f defined on a measurable set E is L-integrable.

5. If f is a bounded measurable function defined on a measurable set E, then prove that $|f|$ is L-integrable over E and also $\left|\int_E f(x)dx\right| \le \int_E |f(x)|\, dx$.

6. If the function f and g are Lebesgue integrable over the measurable set E and if $f(x) < g(x)$ almost everywhere on E, then prove that $\int_E f(x)dx \le \int_E g(x)dx$.

7. Show that the L^p-space is a Normed metric space.

8. Prove that the space L_2 of square summable functions is a linear space.

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