A–1031

Total Pages : 3 **Roll No. ....**

MT-501

M.A./M.Sc. Mathematics (MAMT/MSCMT)

Advanced Algebra-I

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2 \times 19 = 38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *two* (02) questions only.

1. Let H and N are two subgroups of G such that N is normal in G, then prove that $H \cap N$ is a normal subgroup of H and $\frac{H}{H \cap N} \cong \frac{HN}{N}$.

2. Define solvable group. Prove that a group G is solvable iff $G^{(n)} = \{e\}$, for some $n \in N$. Here N is the set of natural numbers.

3. Let R be a Euclidean ring. Let a and b be two non-zero elements of R then prove that :

   (i) if b is unit in R, $d(ab) = d(a)$.

   (ii) if b is not a unit in R, $d(ab) > d(a)$

4. If M_1 and M_2 are two submodules of an R-module M, then prove that :

   (i) $M_1 \cap M_2$ is a submodule of M. and

   (ii) $M_1 + M_2 = \{m_1 + m_2 : m_1 \in M_1, m_2 \in M_2\}$ is a submodule of M.

5. Prove that every finite extension of a field is an algebraic extension but not conversely.

Section–B

Short Answer Type Questions $4 \times 8 = 32$

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *four* (04) questions only.

1. Prove that any two conjugate classes of a group are either disjoint or identical.

2. Define derived subgroup of a group G. Let G be a group then prove that it is abelian iff $G^{(1)} = \{e\}$, e being the identity element in G.

3. Show that the following mapping is linear $t : R^3 \to R^2$ given by $t(x, y, z) = (z, x + y)\ \forall\ (x, y, z) \in R^3$.

4. Define composition series. Prove that an infinite abelian group does not have a composition series.

5. Prove that every field is a Euclidean ring.

6. Prove that every subgroup of a solvable group is solvable.

7. If HK is internal direct product of H and K, then :

$$\frac{HK}{K} \cong H$$

8. If $B = \{e_1 = (1, 0), e_2 = (0, 1)\}$ is the usual basis R^2. Determine its dual basis.

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