A–1016

Total Pages : 3 **Roll No. ....**

MAT-602

M.Sc. Math (MSCMT-23)

Functional Analysis

Examination, 2026 (Feb.)

Time : 2:00 Hrs. Max. Marks : 70

Note* :–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

Section–A

Long Answer Type Questions $2\times19=38$

***Note* :–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. Prove that every normed linear space is a metric space. Converse is not necessary true.

2. Prove that the linear space R^n and C^n of all n-tuples $(x_1, x_2, ..., x_n)$ of real and complex numbers are Banach spaces under the norm $\| x \| = \left\{ \sum_{i=1}^{n} |x_i|^2 \right\}^{1/2}$.

3. Prove that on a finite-dimensional vector space X, any norm $\| \cdot \|$ is equivalent to any other norm $\| \cdot \|_0$.

4. Proof that a compact subset M of a metric space is closed and bounded.

5. Define the following terms :

   (i) Metric space

   (ii) Complete metric space.

   (iii) Pseudo-metric

   (iv) Dense set in metric space

   (v) Every where dense set

   (vi) Nowhere dense

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. Show that the mapping $T : V_2(R) \rightarrow V_3(R)$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(R)$ into $V_3(R)$.

2. Prove that the Euclidean space R^n is a Hilbert space.

3. (i) Explain the Extended Real Number System.

   (ii) State Holder's Inequality for finite sequence.

   (iii) Define the Young's inequality

4. Prove that a convergent sequence in a metric space is a Cauchy sequence.

5. State F. Riesz's lemma.

6. State Zorn's lemma and Hahn Banach Theorem for Real Vector Space.

7. State Open mapping theorem and Bounded Inverse Theorem.

8. State and Proof Banach Contraction fixed point theorem.

****