A–1015

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MAT-601

M.Sc. Math (MSCMT)

Advanced Complex Analysis

Examination February, 2026

Time : 2:00 Hrs. Max. Marks : 70

Note **:–** This paper is of Seventy (70) marks divided into Two (02) Sections 'A' and 'B'. Attempt the questions contained in these Sections according to the detailed instructions given therein. ***Candidates should limit their answers to the questions on the given answer sheet. No additional (B) answer sheet will be issued.***

Section–A

Long Answer Type Questions $2\times19=38$

Note **:–** Section 'A' contains Five (05) Long-answer type questions of Nineteen (19) marks each. Learners are required to answer any *Two* (02) questions only.

1. State and prove the Cauchy integral formula. Furthermore, evaluate the following integral :

$$\int_{\gamma_1} \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where $\gamma_1(t) = \frac{3}{2}e^{it},\ 0 \le t \le 2\pi.$

2. State and prove the Schwarz reflection principle.

3. Classify all the singularities. Show that for an entire function $f : \mathbb{C} \to \mathbb{C}$, ∞ is a pole if and only if f is a polynomial.

4. Evaluate :

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$$

5. Show that any Möbius transformation which maps the unit disc D onto itself is of the form :

$$\varphi(z) = e^{i\theta} \frac{z+\lambda}{1+\bar{\lambda}z},\quad \lambda \in \mathrm{D}, \theta \in \mathbb{R}$$

Section–B

Short Answer Type Questions 4×8=32

***Note* :–** Section 'B' contains Eight (08) Short-answer type questions of Eight (08) marks each. Learners are required to answer any *Four* (04) questions only.

1. State and prove Lioville's theorem.

2. Let $\varepsilon > 0$ and f be holomorphic function in $\{z \in \mathbb{C} : |z| < 1 + \varepsilon\}$ such that whenever $|z| = 1$ we have $|f(z)| > 1$ and $f(0) = i/2$. Show that f has a zero in the unit disc $\{z \in \mathbb{C} : |z| < 1\}$.

3. Find a Möbius transformations that maps 0, 1 ∞ to 1, i, –1 respectively.

4. Show that the zeros of a non-constant holomorphic function are isolated.

5. State and prove the fundamental theorem of algebra.

6. Evaluate the following integral $\int_C \frac{dz}{z-p}$, where C is a closed curve as shown in the following figure, and p is a point inside C.

7. Suppose that $\gamma : [a, b] \to \mathbb{C}$ is a smooth curve and f is continuous on the trace γ^*. If $|f(z)| \leq M$ for all $z \in \gamma^*$ and L is the length of γ then show that :

$$\left|\int_{\gamma} f(z)dz\right| \leq ML$$

8. Show that the function $u(x, y) = e^x \cos y$ is a harmonic function. Find a harmonic function $v(x, y)$ such that $f = u + iv$ is a holomorphic function. Write f in terms of z.

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